Computers & Industrial Engineering 65 (2013) 561–569

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Lot-sizing on a single imperfect machine: ILP models and FPTAS extensions

Kseniya Schemeleva [c], Xavier Delorme [a], Alexandre Dolgui [a,*], Frédéric Grimaud [a], Mikhail Y. Kovalyov [b,a]

[a] Ecole Nationale Supérieure des Mines, FAYOL-EMSE, CNRS UMR 6158, LIMOS, 158, cours Fauriel, 42023 Saint-Etienne Cedex 2, France
[b] United Institute of Informatics Problems, National Academy of Sciences of Belarus, Surganova 6, 220012 Minsk, Belarus
[c] IDRAC, International School of Management, 47, rue Sergent Michel Berthet, CP 607 – 69258 Lyon Cedex 09, France

A R T I C L E   I N F O

Article history:
Received 20 June 2012
Received in revised form 25 February 2013
Accepted 2 April 2013
Available online 25 April 2013

Keywords:
Lot-sizing
Sequencing
Imperfect production
FPTAS
Integer linear program

A B S T R A C T

A single-machine multi-product lot-sizing and sequencing problem is studied. In this problem, items of n different products are manufactured in lots. Demands for products as well as per item processing times are known. There are losses of productivity because of non perfect production. There is also a sequence dependent set-up time between lots of different products. Machine yields and product lead times are assumed to be known deterministic functions. The objective is to minimize the cost of the demand dissatisfaction provided that the total processing time does not exceed a given time limit. We propose two integer linear programming (ILP) models for the NP-hard "fraction defective" case of this problem and compare effectiveness of their ILOG CPLEX realizations with a dynamic programming algorithm in a computer experiment. We also show how an earlier developed fully polynomial time approximation scheme (FPTAS) and one of the ILP models can be extended for a more complex case.

1. Introduction

We study a problem of lot-sizing and sequencing for n different products on a single machine. The finished product can be defective, and thus rejected. Demands for non-rejected (adequate quality) items of all products are known in advance. A sequence-dependent set-up time is required between items of different products to reconfigure the machine. The machine can break down, in which case a repair time is needed. The objective is to find a sequence of lots and their sizes such that the total *demand dissatisfaction cost* is minimized under the condition that the total processing time does not exceed a given time limit T_0. Demand dissatisfaction cost is a linear function of the quantities of unsatisfied demands of all products. We denote this problem as P-Cost.

Motivation for this problem comes from our experience in planning of an automated manufacturing line (Dolgui, Kovalyov, & Shchamialiova, 2011) of several types of Printed Circuit (PC) in electronic industry. This PC production line normally works with no human operators other than the maintenance team. There is no buffers between machines, so this line can be considered as a single machine. A production schedule is made for 24 h to satisfy given 1-day demands, and it is repeated every day. It specifies lot sizes for all PC types and a sequence of lots. The main difficulties are the high defectiveness rate – from 20% to 40% depending on the type of the PC, and the line breakdowns – mean time to failure for different pieces of equipment varies between 100 and 1000 h and mean time to repair is about half an hour. A similar situation is described in Sikora, Chhajed, and Shaw (1996).

A time diagram for the flow of items at the machine output is given in Fig. 1. There is set-up times between lots of products including a set-up time before a first lot. Normal processing is interrupted by a single breakdown for each of two lots represented (the first lot is in green and the last lot is in blue), there is one defective item in the first lot and two defective items in the last lot. As the planning horizon is limited by T_0 (24 h in the industrial case), the time is not enough to finish the last lot, so two items will not be processed (items with lateness). The quantities of good items for all lots are compared with demands, the dissatisfaction cost will be incurred for the lots were demand is greater than the quantity of good items.

Problem P-Cost belongs to the general field of lot-sizing and scheduling. A survey of *deterministic* lot-sizing and scheduling problems can be found in Drexl and Kimms (1997). In Potts and Kovalyov (2000) a review of scheduling with batching was given. Papers of Zhu and Wilhelm (2006) as well as Allahverdi, Ng, Cheng, and Kovalyov (2008) contain literature on scheduling with set-up times and costs. An earliness-tardiness lot-sizing and scheduling problem to minimize the sum of setup, holding and tardy costs was studied in Supithak, Liman, and Montes (2010). Additional

* Corresponding author. Tel.: +33 4 77 42 01 66; fax: +33 4 77 42 66 66.
E-mail addresses: kseniya.schemeleva@idraclyon.com (K. Schemeleva), delorme@emse.fr (X. Delorme), dolgui@emse.fr (A. Dolgui), grimaud@emse.fr (F. Grimaud), kovalyov_my@newman.bas-net.by (M.Y. Kovalyov).

0360-8352/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cie.2013.04.002

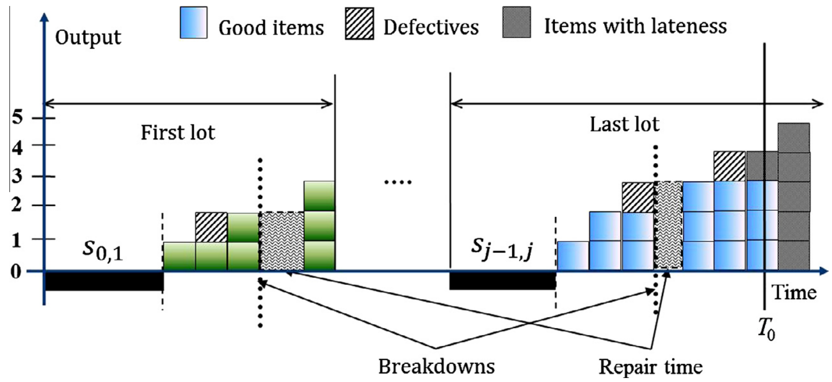

Fig. 1. An item flow time diagram for line output.

results for the deterministic lot-sizing and scheduling problems can be found in Drexl and Haase (1995), Sikora (1996), Meyr (2002), Toso, Morabito, and Clark (2009), Dolgui and Proth (2010) and Dolgui, Eremeev, Kovalyov, and Kuznetsov (2010).

Our model assumes that the machine can produce defective items and can break down. Lot-sizing and scheduling problems with uncertain product lead time and machine yield are also considered in a number of papers. The most cited survey of lot-sizing problems with random yields was given by Yano and Lee (1995). A number of lot-sizing problems with random yield were reviewed by Grosfeld-Nir and Gerchak (2004). Lead time uncertainty for assembly systems were investigated by Dolgui and Ould-Louly (2002) and Ould-Louly and Dolgui (2004). A state of the art for the supply planning under uncertainties in the case of random demand and lead time in a Manufacturing Resource Planning environment is given by Dolgui and Prodhon (2007). A method to model machine breakdowns using a renewal process can be found in (Dolgui, 2002).

A problem most similar to ours is studied by Dolgui, Levin, and Louly (2005), where the authors suppose that random yield can be characterized by Bernoulli's law and machine breakdowns can be modeled using renewal process. Authors decompose their NP-hard problem into three sub-problems and propose a dynamic programming algorithm to solve the lot-sizing part of the original problem.

In this paper, in contrast, we mainly consider the "fraction defective" case, in which f_i approximates an average proportion between defective and adequate quality items of product i, and $R(x)$ approximates an average proportion between total machine repair time and total manufacturing time. In this case, functions f_i and R can be easily deduced from classic process indicators such as defectiveness rates, mean time to failure and mean time to repair. The user can also integrate in these functions some additional safety values. The "fraction defective" assumption is widely used in the literature, see Inderfurth, Lindner, and Rachaniotis (2005), Teunter and Flapper (2003), Inderfurth, Janiak, Kovalyov, and Werner (2006), Inderfurth, Kovalyov, Ng, and Werner (2007), Chiu, Ting, and Chiu (2007) and Buscher and Lindner (2007). Flapper, Fransoo, Broekmeulen, and Inderfurth (2002) underline that uncertainty about the distribution of the defective items is common for many process industries. Nevertheless, if there is an adequate statistical data, it is possible to accurately determine that the rate of defective items is relatively stable and fixed.

The following parameters are assumed to be given:

d_i – demand for the good items of product i, $i = 1,\dots,n$;
t_i – per item processing time of product i, $i = 1,\dots,n$;
c_i – per item cost of demand dissatisfaction for product i, $i = 1,\dots,n$;
$s_{i,j}$ – set-up time for a pair of products i, j, if a lot of product i immediately precedes a lot of product j, $s_{i,j} \geqslant 0$;
$s_{0,i}$ – initial set-up time, if the manufacturing process begins with a lot of product i, $i = 1,\dots,n$, $s_{0,i} \geqslant 0$;
$f_i(x_i)$ – an integer non-decreasing function whose value is equal to the estimated quantity of defective items in a lot of size x_i of product i. We require $f_i(0) = 0$, and $f_i(x_i) \leqslant x_i$;
$R(x)$ – non-negative non-decreasing real valued function which represents the forecasted cumulative repair time for the lot size vector $x = (x_1,\dots,x_n)$.

All d_i, t_i and c_i are integer positive numbers. We assume that the set-up times satisfy the triangle inequality: $s_{i,j} + s_{j,k} \geqslant s_{i,k}$ for any triple of products (i,j,k), where $i = 0,\dots,n$, $j,k = 1,\dots,n$ and $i \neq j \neq k$. Note that the set-up time matrix may not be symmetric, i.e., $s_{i,j} \neq s_{j,i}$ may happen for $i,j \in \{1,\dots,n\}$. Sequence-dependent set-ups are widely studied for lot-sizing and scheduling problems, see, for example, Shim, Kim, Doh, and Lee (2011).

There are some additional assumptions:

- the machine can process at most one item at a time;
- item processing is not possible during set-up or repair time;
- set-up and repair times cannot overlap;
- breakdown can only happen during item processing;
- items of the same product pass through the machine with no time delay between them (the transfer time is included in the processing time).

The deterministic problem P-Cost considered in this paper was originally proposed in Dolgui et al. (2011), where the authors prove that its lot-sizing part, called P-Cost-Sizes, is NP-hard even in the "fraction defective" (FD) case and propose a dynamic programming algorithm and a fully polynomial time approximation scheme (FPTAS). In the present paper, we show that the problem P-Cost-Sizes-FD admits two integer linear programming formulations (ILP). Further we compare the performances of the dynamic programming algorithm and two ILP formulations solved by ILOG CPLEX. Finally, we demonstrate that the FPTAS and one of the ILP models can be transformed to solve a more general than "fraction defective" case, denoted as P-Cost-Sizes-$\alpha\beta$.

The remainder of this paper is organized as follows. In Section 2, we provide a mathematical model and some observations about the problem P-Cost. Section 3 describes two ILP models for the "fraction defective" case of this problem. Section 4 contains a comparison of the ILOG CPLEX realizations of the proposed models with the earlier developed dynamic programming algorithm. In Section 5, we introduce the problem P-Cost-Sizes-$\alpha\beta$ and demonstrate the transformations required for the FPTAS and one of the ILP models to solve this new problem. Some conclusions are reported in Section 6.