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Dynamic lot sizing for a warm/cold process: Heuristics and insights

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ABSTRACT

We consider the dynamic lot sizing problem for a warm/cold process where the process can be kept warm at a unit variable cost for the next period if more than a prespecified quantity has been produced. Exploiting the optimal production plan structures, we develop nine rule-based forward solution heuristics. Proposed heuristics are modified counterparts of the heuristics developed previously for the classical dynamic lot sizing problem. In a numerical study, we investigate the performance of the proposed heuristics and identify operating environment characteristics where each particular heuristic is the best or among the best. Moreover, for a warm/cold process setting, our numerical studies indicate that, when used on a rolling horizon basis, a heuristic may also perform better costwise than a solution obtained using a dynamic programming approach.

1. Introduction

In this paper, we consider the problem of dynamic lot sizing for a special type of production processes. The dynamic lot sizing problem is defined as the determination of the production plan which minimizes the total (fixed setup, holding and variable production) costs incurred over the planning horizon for a storable item facing known demands.

Recently, the notion of a "warm/cold process" has been introduced into the scheduling literature (Toy and Berk, 2006). A warm/cold process is defined as a production process that can be kept *warm* for the next period if a minimum amount (the so-called warm threshold) has been produced in the current period and would be *cold*, otherwise. Production environments where the physical nature of the production technology dictates that the processes be literally kept warm in certain periods to avoid expensive shutdown/startups are typical in glass, steel and ceramic production. Robinson and Sahin (2001) provide other examples in food and petrochemical industries where certain cleanup and inspection operations can be avoided in the next period if the quantity produced in the current period exceeds a certain threshold. Production processes where production rates can be varied also fall into the warm/cold process category. The upper bound on the production rate is the physical capacity of the production process and the lower bound corresponds to the warm threshold, below which the process cannot be kept running into the next period without incurring a setup. Such variable production rates can be found in both discrete item manufacturing and process industries. Change in production rate can be obtained at either zero or positive cost depending on the characteristics of the employed technology. The additional variable cost is, then, the variable cost of keeping the process warm onto the next period.

As the above examples illustrate, the dynamic lot-sizing problem in the presence of production quantity—dependent warm/cold processes is a common problem. This problem, in the presence of no shortages, has been formulated and solved optimally by Toy and Berk (2006) using a dynamic programming approach with an $O(N^3)$ forward algorithm where N denotes the problem horizon length. Later, they extend their results to the case where some of the demands may be lost under a profit maximization objective (Berk et al., 2008).

The dynamic lot sizing problem for a warm/cold process is a generalization of the so-called classical problem which was first analyzed by Wagner and Whitin (1958). The classical problem assumes uncapacitated production and no shortages. Wagner and Whitin (1958) provide a dynamic programming solution algorithm and structural results on the optimal solution. Their fundamental contribution lies in establishing the existence of planning horizons, which makes forward solution algorithms possible. Although the optimal solution structure is known, the complexity of obtaining it (shown to be $O(N \log N)$ in general by Federgruen and Tzur, 1991; Wagelmans et al., 1992; Feng et al., 2011 for constant capacities) has stimulated a stream of research that focuses on developing lot sizing heuristics based on simple stopping rules, such as Silver–Meal (Silver and Meal, 1973), Part-Period Balancing (DeMatteis, 1968), Least Unit Cost, Economic Order Interval, McLauren's Order Moment (Vollmann et al., 1997), Least Total Cost (Narasimhan and McLeavy, 1995), Groff's

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Algorithm (Groff, 1979). (See also Sahin et al., 2008; Narayanan and Robinson, 2010.)

Further results on the lot sizing problem are found in the literature on its extension to the capacitated production settings. The capacitated lot sizing problem (CLSP) is related to the lot sizing problem for a warm/cold process under certain conditions (see Toy and Berk, 2006). The CLSP has been first studied by Manne (1958) and has been shown to be NP-hard by Florian et al. (1980). Reviews of the works on CLSP (along with the uncapacitated versions) are by Brahimi et al. (2006) and Quadt and Kuhn (2008), who include extensions of the problem, and Buschkühl et al. (2010). Recent analytical studies have focused on novel solution approaches. Heuvel and Wagelmans (2006) develop an $O(T^2)$ algorithm. Pochet and Wolsey (2010) provide a mixed integer programming reformulation that can be solved with LP-relaxation to optimality under reasonable conditions. Chubanov et al. (2008) and Ng et al. (2010) introduce polynomial approximations. Hardin et al. (2007) analyze the quality of bounds by fast algorithms. Reviews of meta-heuristic approaches to the CLSP can be found in Staggemeier and Clark (2001), Jans and Degraeve (2007) and in Guner Goren et al. (2010) on genetic algorithms for lot sizing. A recent review of related works appears also in Glock (2010).

Rule-based heuristics in rolling horizon environments have been studied by Stadtler (2000), Simpson (2001), and Heuvel and Wagelmans (2005). The work herein joins this stream by considering the dynamic lot sizing problem for a warm/cold process. Specifically, we propose rule-based lot sizing heuristics for the problem and examine the efficacy of such rules. To the best of our knowledge, this is the first work that studies lot sizing rules for the operating environment where the production process can be kept warm at some cost if production quantity in a period exceeds a threshold value. We believe that our contributions lie in developing a number of heuristics which perform well in certain operational environments and in identifying such regions for selecting a particular heuristic. We consider the application of the proposed heuristics in a static setting as well as on a rolling horizon basis as it is the practice. The available commercial ERP software (*e.g.*, *SAP*) still offer well-known heuristics for the classical lot sizing problem as options for decision-makers along with the 'optimal' solution algorithms in their manufacturing modules. For the conventional production environments, the benefits of heuristics include the ease of use, smoother production schedules and more intuition for the trade-offs. Moreover, for a warm/cold process setting, our numerical studies indicate that, when used on a rolling horizon basis, a heuristic may also perform better costwise than a solution obtained using a dynamic programming approach. This finding is consistent with similar studies on the classical problem (Stadtler, 2000; Heuvel and Wagelmans, 2005). Hence, investigation of heuristics for warm/cold process settings may be financially beneficial in practice as well as from a purely theoretical perspective. Our work extends the heuristics literature on the dynamic lot sizing problem.

The rest of the paper is organized as follows: In Section 2, we introduce the basic assumptions of our model, formulate the optimization problem and present some key results. In Section 3, we present some theoretical results on an economic production quantity (EPQ) model that we use as a continuous counterpart of a warm/cold process to develop some of our heuristics. In Section 4, we introduce and construct nine lot sizing heuristics for a warm/cold process. In Section 5, we present a numerical study and discuss our findings in regards to the cost performance of the proposed heuristics. In our numerical study, we provide results on the performance distribution of individual heuristics, on the rankings of the heuristics, on identifying the operating environment where a particular heuristic may perform best and on the impact of planning horizon lengths when production plans are made and executed on a rolling horizon basis.

2. Model: assumptions and formulation

We consider the operational setting in Toy and Berk (2006) with time-invariant system and cost parameters. We assume that the length of the problem horizon, N is finite and known. Demand in period t, denoted by D_t $(t=1,2,\ldots,N)$, is non-negative and known, but may be different over the problem horizon. No shortages are allowed; that is, the amount demanded in a period has to be produced in or before its period. The amount of production in period t is denoted by x_t. If $x_t>0$, the production indicator δ_t is 1, zero otherwise. The inventory on hand at the end of period t is denoted by y_t $(=y_{t-1}+x_t-D_t)$. Inventory holding cost per unit of ending inventory is h per period. Without loss of generality, we assume that the initial inventory level is zero. We assume that unit production cost is c but may be omitted in the analysis since all demands must be met over the horizon.

Production quantity in a period cannot exceed the capacity, R. For feasibility, we assume that, for any t, there exists a $j(t)$ for which $\sum_{i=t}^{j(t)} D_i \le (j(t)-t+1)R$ for $t\le j(t)\le N$, $1\le t\le N$. This condition guarantees that any subset of demands can be produced within the horizon; a special case of the condition is satisfied when $D_t \le R$ for all t. We consider a warm/cold production process: The production process may be kept *warm* onto the beginning of period t if the production quantity in the previous period is at or above a threshold value Q; that is, $x_{t-1}\ge Q$. Otherwise, the process cannot be kept warm and is *cold*. Let z_t indicate the warm/cold status of the process as period t starts; it attains a value of 0 if the process is warm and 1, otherwise. In order to keep the process warm onto period t, warming cost ω is charged for every unit of unused capacity in period $t-1$. That is, the warming cost incurred in period $t-1$ would be $\omega(R-x_{t-1})$ monetary units. Note that, even if the quantity produced in period $t-1$ is at least Q, it may not be optimal to keep the process warm onto the next period if there would not be any production during the next period. In such instances, there will be no warming costs incurred although $x_{t-1}\ge Q$ since $x_t=0$. We assume that a warm process requires no setup (and, hence incurs no setup cost) but a cold process requires a cold setup with a fixed cost K (>0) if production is to be done in the period. Finally, we assume that $h>\omega$ which ensures the Wagner–Whitin type cost structure, and that the warm/cold process threshold is between the point of indifference and the capacity, $R-(K/\omega)<Q\le R$. (For the implications of these assumptions, see Toy and Berk, 2006.)

The objective is to find a production plan $x_t\ge 0$ $(t=1,2,\ldots,N)$ (timing and amount of production), such that all demands are met at minimum total cost over the horizon. Let $X=\{x_1,\ldots,x_N\}$ denote a feasible production plan constructed over periods 1 through N; Γ_t be the variable cost incurred within period t computed as $\Gamma_t=hy_t+\omega(R-x_t)\delta_{t+1}(1-z_{t+1})$ under the given production plan; and THC denote the total horizon cost. Then, the optimization problem (P) can be formally stated as follows:

$$\min_X THC=\sum_{t=1}^{N}(K\delta_t z_t+\Gamma_t)$$

subject to $0\le x_t\le R$, $y_{t-1}+x_t\ge D_t$, for all t; $z_1=1$, $\delta_{N+1}=0$, $z_{N+1}=1$, and $y_0=y_N=0$.

Let $L_{u,v}$ represent a subset of X between periods u and $v-1$ (inclusive) such that the starting inventory in period u and ending inventory in period $v-1$ are zero and production is done in all periods u through m to cover the demands for periods u through $v-1$. Formally, $L_{u,v}=\{x_t|x_t>0, t=u,\ldots,m;\ x_t=0 \text{ for } t=m+1,\ldots,v-1;\ y_{u-1}=y_{v-1}=0\}$ for $0\le u\le m<v\le N+1$. With a slight abuse of