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# Information precision and asymptotic efficiency of industrial markets

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**Abstract**

I present a strategic model of a bilateral oligopoly with asymmetric information to examine (i) the validity of the conjecture of price-taking behavior in such markets as the number of agents becomes large and (ii) the effect of the rate that individual information precision decreases with increased number of agents on convergence to price-taking and efficiency. I show that with downstream competition, increasing the number of sellers may make all participants price-takers in the limit, but increasing the number of buyers may not. When the total precision of information in the market is high, price-taking and full social efficiency is achieved in the limit with large numbers of buyers and sellers. However, if the total precision of information in the market is poor, price-taking conjecture may fail and large inefficiencies, including full inefficiency, can occur in the limiting outcome. The rate of decrease of individual information precision with increased number of agents determines the rate of convergence to efficiency, and the convergence is slower than that predicted by single-unit auction models in the literature. I also demonstrate that when the number of sellers or both the number of buyers and the sellers go to infinity, price-taking and information aggregation tend to go together. When the number of buyers goes to infinity, however, information can get aggregated when the agents do not become price-takers in the limit. Albeit, in the latter case, the aggregated information is masked by the noise in the sellers' signals and the cost variability.


*JEL Classification:* D82; L10; L13

*Keywords:* Private information; Bilateral oligopoly; Large markets; Price-taking

## 1. Introduction

Many models of economies with large number of agents assume price-taking behavior. This assumption is usually based on the intuition that when the agents are small compared to the size of the economy, their individual trade sizes are so small that the price impact is negligible. A conceptually distinct support for the price-taking assumption is that increasing the number of agents in the economy makes them so competitive that even small changes in their "quoted" price causes dramatic declines in the quantities they can sell or buy, i.e., their demand or supply curves as a function of price are nearly vertical and they are nearly true price-takers. In this paper, I present a strategic

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model of a vertical industrial market with upstream and downstream competition to question the validity of both of these arguments under private information and show that neither of these two arguments has to hold when there is a large number of agents. In particular, I show that the specific way the precision of total information in the economy changes with the increased number of agents, together with the payoff structure that characterizes such markets can have important consequences for price-taking and efficiency as the number of agents in the market becomes large.

To demonstrate the effect of private information on price-taking behavior in industrial markets, I build and analyze a model of vertical trade in an industrial setting: there are $N$ manufacturers who compete as a Cournot oligopoly in a consumer market with uncertain demand. $M$ upstream suppliers provide a homogenous intermediate good via identical constant marginal cost technologies. The realization of the marginal cost coefficient is uncertain, reflecting the variation on the price of certain common inputs (for instance oil, energy or raw materials). The agents have private (but correlated) signals about the relevant uncertainty that they are facing. Specifically, each buyer has an imperfect signal about the realization of the consumer demand, and each seller has an imperfect signal about the realization of the marginal cost.

To explore the emergence and nature of price-taking behavior in this market, we have to endogenize price taking through a strategic model. To do this, I employ a common approach and use demand and supply curves as the strategies of the participants (see, e.g., Kyle, 1989; Vayanos, 1999). This results in each participant's facing a downward sloping residual demand curve or an upward sloping residual supply curve when making her quantity decision. Therefore, for each agent, increasing the quantity he sells (buys) decreases (increases) the market price. The lower the price-impact of the trades for an agent, the closer she is to being a price-taker. This constitutes a concrete measure of price taking, namely, the price impact of the trades, whose magnitude emerges endogenously in equilibrium. As a result, (near) perfectly elastic supply where all agents are price-takers in the limit and (near) perfectly inelastic supply where trading even the smallest quantities can have arbitrarily large price impact in the limit and everything in between can arise endogenously in equilibrium as the number of agents in the market becomes large.

A key issue is the trade-off between two forces that affect the price-taking behavior of the agents: first, consistent with the common wisdom, as the number of sellers increases, competition forces the supply functions of the sellers to become steeper. This effect not only pushes the sellers towards being price-takers, but also, since their residual supply curves (as functions of the price) become steeper, pushes the buyers towards being price-takers. The second and the opposing force is the effect of information aggregation in the market and the agents' reaction to that when determining their optimal quantities conditional on the realization of the market outcome. If a seller puts a high quantity in the market, this signals her willingness to produce and the increased possibility of her having a low cost signal. This increases the remaining sellers' belief that the cost realization will be low. As a consequence, the prices are pushed lower. The higher the magnitude of this adverse selection effect the farther the suppliers will be pushed away from being price-takers. The growth rate of the magnitude of this effect as the number of suppliers increases compared to the rate that the competition pushes the suppliers towards being price-takers determines the price-taking behavior in the limit. A symmetric argument applies for the buyers.

The reaction of the participants to the market outcome is determined by their expectations on the behavior of the other agents in the market. When the private information that each of the remaining agents possesses is relatively accurate, each one of them will respond to her signal by reacting strongly to it. That is when a buyer has an accurate signal about consumer demand, she will increase her quantity to buy by a sizeable amount when she receives a high signal and vice versa, and therefore her trading quantity has a sizable ex ante variance. On the other hand, when a trader's signal is not very accurate, she does not respond to it strongly, and her trading quantity becomes more predictable to the other agents. Aggregated over all agents, the residual supply or demand curve that each agent is facing will be more predictable if the accuracy of the signals is low. As a consequence, any deviation will signal a strong realization of the underlying variable and result in a strong reaction by the remaining agents. In the limit, this can become so extreme that smallest quantities can have very large impact on prices, and consequently, the market performance can suffer dramatically.

Here, the way individual information precision decreases with the number of agents becomes important. Intuitively, one could argue that as the number of competitors in a market increases, their incentives to acquire more precise information along with the gains from using such precise information decreases. Given this premise, depending on the cost function for information acquisition, the total information in the market can increase, stay the same or decrease