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# Joint economic lot sizing problem for a three—Layer supply chain with stochastic demand

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ABSTRACT

This paper considers the joint economic lot-sizing problem (JELP) for multi-layer supply chain with multi-retailers and single manufacturer and supplier. The paper extends the work of (Ben-Daya et al., 2013) and relaxes the assumption of deterministic demand and constant holding and ordering costs. The paper proposes modifying four computational intelligent algorithms to solve mixed integer problems and compares their performance for solving the problem at hand. The paper, further, compares between adopting a centralized safety stock policy versus a decentralized policy. Results showed cuckoo search to outperform other tested algorithms and, also, favored adopting a centralized policy.



## 1. Introduction

The unprecedented competitive business environment in the past twenty years has brought supply chain management to the surface as one of the most influential strategies used for enhancing organizational competitiveness (Gunasekaran et al., 2008; Moncayo-Martínez and Zhang, 2013, 2011). Organizations are not only seeking an enhanced internal operational efficiency but also coordinating with their customers and suppliers for the efficient management of inventories across the supply chain (Sari et al., 2012). With this regard, all parties are seeking an economic order quantity (EOQ) based-policy that ensures the fulfillment of customers' demand while minimizing their integrated total cost function (Ben-daya and As'ad, 2009; Harris, 1913). Such a problem is generally called the joint economic lot-sizing problem (JELS) (Sari et al., 2012). This problem has been the focus of a major stream of articles in the supply chain body of knowledge starting with Goyal (1977) to recent ones such as Ben-Daya et al. (2013). The reader is kindly referred to Ben-Daya et al. (2008), Glock (2012a) for a comprehensive review on the problem.

Recent literature shows a noticeable number of articles that aimed to develop mathematical models addressing this class of problems with a primary objective of identifying the optimal coordinated production and shipment policy with a minimum chain-wide total cost (Ben-daya and As'ad, 2009). The vast majority of these models were limited to two layer supply chains with deterministic constant demand and with single-vendor single-buyer (Ben-daya and As'ad, 2009; Glock, 2012b).

Building on the work of Ben-daya and As'ad (2009), an enhanced optimization model of JELS was proposed in Ben-Daya et al. (2013) for a three layers network with a single supplier, a single manufacturer, and multi-retailers. Getting the optimal solution for NLIP models using traditional mathematical methods is difficult even by using specialized software as it will be computationally expensive (Cárdenas-Barrón et al., 2012). Thus, the model was solved by relaxing all integer decision variables to continuous variables to convert the model from non-linear integer programming (NLIP) to a non-linear programming (NP).The authors, then, used algebraic methods to drive a near optimal solution and developed an algorithm to get the integer values for those relaxed decision variables. Some mathematical shortcoming in the model was later corrected in Cárdenas-Barrón et al. (2012) and an algorithm was proposed and proved to find the integer values more efficiently and effectively with respect to CPU time than the algorithm developed in Ben-Daya et al. (2013).

To bear a better resemblance to practice, this paper studies the effect of relaxing the assumptions of deterministic demand and the assumption of holding and ordering costs equality among all retailers as was in the former models. Besides solving the new problem using derivative-free method, the paper proposes,

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implements, and compares four bio-inspired algorithm for solving this problem given its stochastic nature.

Following the introduction section, the rest of this paper is organized as follows. Section 2 defines the problem under consideration and the proposed model assumptions. The mathematical model is developed in Section 3 and model analysis methods are provided in Section 4. Numerical analysis are presented in Section 5 and, finally, concluding remarks are given in Section 6.

## 2. Problem definition

This paper builds on the joint economic lot sizing problem proposed in Ben-Daya et al. (2013) with the objective of minimizing the total cost across a supply chain network by determining the optimal policy for the ordering, production, and shipment lot sizing. The problem considers a three-layer make-to-stock (MTS) supply chain network with one supplier, one manufacturer, and multi-retailers. For a given lot, shipments are allowed to take place during the production, and finished products are shipped in equal shipments to retailers who receive these at the same time. Shipments' size, however, may vary from one retailer to another depending on each retailer's demand.

Ben-Daya et al. (2013) proposed model assumed: (1) deterministic and constant demand; (2) fixed ordering and holding cost for all retailers; (3) inventory holding costs are increasing in the downstream direction; (4) the cycle time of the supplier is an integer multiplier of that for the manufacturer, which in turn is an integer multiplier of the cycle time of the retailers; and (5) the supply chain is vertically integrated, so that system-wide optimization is acceptable to all parties involved and benefit sharing among supply chain members is not an issue (Glock, 2012b). This paper considers the same problem but relaxes the first two assumptions, in other words: tolerating for stochastic demand, and for varying ordering and holding costs among retailers.

## 3. Mathematical model development

The proposed model tolerates varying ordering and holding costs for all retailers and assumes stochastic demand (follows a normal distribution with mean $\mu$ and standard deviation $\sigma$) and these parameters vary for each stage. Given that the demand is stochastic, there will be a need for a safety stock quantity at every inventory in the supply chain to decrease the running-out-of-stock that affects customer satisfaction and reliability (Ben-Daya and Hariga, 2004; Chopra and Meindl, 2007; Glock, 2012b; Romeijn et al., 2007). This additional quantity will typically affect the inventory holding cost that, in turn, will affect the total cost of the chain.

Safety stock placement decision in a multi-layer supply chain network has been the focus of many research articles (e.g. Ballou and Burnetas (2003), Duan and Liao (2013), Zinn et al. (1989). The major issue governing such decision situation is the trade-off between maintaining minimum safety stock levels and satisfying the end customer service level (Osman and Demirli, 2012).

A centralized control policy is usually better than decentralized control based on unit system costs achieved but the magnitude of the savings will vary with demand patterns (Duan and Liao, 2013). In cases of lower savings, thus, decentralization represents an opportunity to improve customer service (Wanke, 2009). The placement decision is also affected by whether the members of the supply chain belong to the same organization or not. In the latter case, saving-sharing mechanisms need to be put into action as an incentive for retailers (Baboli et al., 2011; Duan and Liao, 2013).

From a conceptually and an administratively attractive vantage point, many firms prefer a decentralized control policy that allows individual divisions to make their own inventory decisions (Cattani et al., 2011). Mechanisms for such policy still receive attention in the literature (e.g., Caron and Marchet (1996), Cattani et al. (2011)). This paper will consider both policies; centralized and decentralized safety stock leaving the door opened for future research. The mathematical model, thus will be developed considering the general case; decentralized safety stock.

**Table 1**
Mathematical notations.

| **Mathematical notations** | |
|---|---|
| $P_s$ | Production rate of the supplier |
| $P_m$ | Production rate of the manufacturer |
| $\mu_s$ | Supplier's average demand rate |
| $\mu_m$ | Manufacturer's average demand rate |
| $\mu_{rj}$ | Average demand rate observed by retailer $j$, $\mu_s=\mu_m=\sum_{j=1}^{n_r}\mu_{rj}=\mu$ |
| $\sigma_s$ | Demand standard deviation of the supplier |
| $\sigma_m$ | Demand standard deviation of the manufacturer |
| $\sigma_{rj}$ | Demand standard deviation observed by retailer $j$, $\sigma_s=\sigma_m=\sum_{j=1}^{n_r}\sigma_{rj}=\sigma$ |
| $F_s$ | The parameter that controls the corresponding service level at the supplier |
| $F_m$ | The parameter that controls the corresponding service level at the manufacturer |
| $F_{rj}$ | The parameter that controls the corresponding service level at retailer $j$ |
| $T_s$ | Supplier's cycle time |
| $T_m$ | Manufacturer's cycle time |
| $T$ | Common basic cycle time for all retailers |
| $t_0$ | The setup and transportation time for the supplier's raw material |
| $t_s$ | The setup and transportation time for the supplier's finished product and manufacturer's raw material |
| $t_m$ | The setup and transportation time for manufacturer's finished product |
| $k_1$ | Integer multiplier of the manufacturer's cycle time, $T_s=k_1\,T_m$ ($T_s=k_1\,k_2T$) |
| $k_2$ | Integer multiplier of the retailers' cycle time, $T_m=k_2T$ ($T_s=k_1\,k_2T$) |
| $h_0$ | The per unit holding cost for the supplier's raw materials per unit time |
| $h_s$ | The per unit holding cost for the supplier's finished product and manufacturer's raw materials per unit time |
| $h_m$ | The per unit holding cost for the manufacturer's finished products per unit time |
| $h_{rj}$ | The per unit holding cost for retailer $j$ per unit time |
| $A_s$ | Supplier's production setup cost |
| $A_m$ | Manufacturer's production setup cost |
| $O_s$ | Supplier's raw material ordering cost |
| $O_m$ | Manufacturer's raw material ordering cost |
| $O_{rj}$ | Retailer $j$ finished product ordering cost |
| $m_1$ | Number of raw material shipments received by the supplier within a cycle |
| $m_2$ | Number of raw material shipments received by the manufacturer within a cycle |
| $n_r$ | Number of retailers |
| $Q_s$ | The lot size received and produced by the supplier per cycle |
| $G_s$ | The shipment size received by the supplier, $G_s=Q_s/m_1$ |
| $Q_m$ | The lot size received and produced by the manufacturer per cycle |
| $G_m$ | Shipment size received by the manufacturer, $G_m=Q_m/m_2$ |
| $G_r$ | Shipment size received by all retailers |
| $G_{rj}$ | Shipment size received by the retailers $j$, $G_r=\sum_{j=1}^{n_r}G_{rj}$ |
| $TC_s$ | Total cost per unit time for the supplier |
| $TC_m$ | Total cost per unit time for the manufacturer |
| $TC_r$ | Total cost per unit time for all retailers |
| $TC(k_1, k_2, T, m_1, m_2)$ | Total cost per unit time for the whole supply chain |