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The capacitated Lot Sizing model: A powerful tool for logistics decision making

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ABSTRACT

Starting from the seminal intuitions that led to the developments of the Economic Order Quantity model and of the formulation of the Dynamic Lot Sizing Problem, inventory models have been widely employed in the academic literature and in corporate practice to solve a wide range of theoretical and real-world problems, as, through simple modifications to the original models, it is possible to accommodate and describe a broad set of situations taking place in complex supply chains and logistics systems.

The aim of this paper is to highlight, once more, the powerfulness of these seminal contributions by showing how the mathematical formulation of the Capacitated Lot Sizing Problem can be easily adapted to solve some further practical logistics applications (mainly arising in the field of coordination of transportation services) not strictly related to manufacturing and production environment. Mathematical formulations and computational experiences will be provided to support these statements.

1. Introduction

The history of inventory problems can be rooted back to the Economic Order Quantity (EOQ) model presented by Harris (1913), also known as the Wilson Lot Size formula, since it was firstly used in practice by Wilson (1934). The EOQ model assumes the presence of a single item whose demand is continuous (with a constant known rate) and an infinite planning horizon. The solution of the model is easy and provides the optimal quantity to be ordered, balancing the setup and inventory holding costs. However, with the same assumptions, in presence of multiple items and capacity restrictions the model becomes NP-hard (Hsu, 1983). The Dynamic Lot Sizing Problem (in the following, generically referred to as DLSP or Lot Sizing), first proposed by Wagner and Whitin (1958), can be considered as an extension of the EOQ model. In this new version, on a discrete time scale, deterministic dynamic demand and finite time horizon are considered while the objective function is the same basic trade-off between setup and inventory costs.

Starting from these seminal papers, further variants of the problem have been introduced. These are mainly concerned with the extension to the multi-item case (Barany et al., 1984), the introduction of several conditions about the costs, limitations on production capacities (leading to the Capacitated Lot Sizing Problem, in the following CLSP) (Bitran and Yanasse, 1982) and possible additional features regarding, for instance, demand uncertainty (Brandimarte, 2006), setup costs and/or times (Trigeiro et al., 1989), linked lot sizes (Suerie and Stadtler, 2003), alternative suppliers (Basnet and Leung, 2005). Combinations of these aspects can provide models with very different complexities.

Interesting reviews about models and methods to tackle Lot Sizing problems have been published by Kuik et al. (1994), Drexl and Kimms (1997), Karimi et al. (2003), and Jans and Degraeve (2008), while a rich textbook on the topic has been provided by Pochet and Wolsey (2006).

Jans and Degraeve (2008) compiled a very interesting and complete survey devoted to describe actual and potential variants of the problem. The authors highlight how most of them are inspired by specific real life applications and, in particular, they focus on a variety of industrial production planning problems. The application of the Lot Sizing model, and its variants, to real-world problems constitute a very active research strand (see, for instance: Rezaei and Davoodi, 2011; Ferreira et al., 2012; Liao et al., 2012).

In this paper we want to show how, through an appropriate interpretation of the elements of the model, Lot Sizing formulations can also be effectively used to face further practical logistic problems, outside of the classical field of production and manufacturing planning. Therefore, rather than providing original

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models, the aim of the paper is to show how standard formulations can be used to support decisions in other contexts of applications; in this sense, the more established these models are, the more powerful and insightful will be their adaptation, as existing results in terms of formulations and solution approaches can be easily exploited.

The remainder of this paper is arranged as follows. In the next section we introduce the mathematical model of the CLSP, considering the single item and the multi-item versions. Then we illustrate a general framework indicating how these models can be used to describe different kinds of logistic problems. In particular three specific examples are introduced and discussed: the optimization of the departure schedule for a bus terminal; the management of a logistic cross-dock platform; and the optimization of an airport check-in gates configuration. For the above problems, we explain how they can be formulated, through few adaptations, starting from the CLSP model. Furthermore, some case studies (related to real-world situations) are presented, showing how these models can be solved in limited computational times and be used as decision support tools. Finally, some concluding remarks and directions for future research are drawn.

2. Mathematical models for Lot Sizing problems: generalities

By denoting with $t \in \{1..N\}$ one of the N time buckets introduced to divide the planning horizon, the following parameters can be considered, referred to the specific time period t and to a single product scenario:

- d_t: the demand forecast;
- p_t: the unit production or purchasing cost;
- h_t: the unit inventory cost;
- f_t: the fixed setup or ordering cost; and
- C_t: the maximum feasible lot size (capacity).

Introducing the variables

- s_t: stock at the end of period t;
- x_t: quantity to be produced or ordered during period t;
- y_t: binary variable equal to 1 if units of the product are manufactured (or ordered) in period t (0 otherwise).

The DLSP can be formulated as follows:

$$\min z = \sum_{t=1}^{N} (p_t x_t + h_t s_t + f_t y_t) \quad (1)$$

s.t.

$$s_t = s_{t-1} + x_t - d_t \quad t = 1,..,N \quad (2)$$

$$s_t = 0 \quad t = 0 \text{ and } t = N \quad (3)$$

$$x_t \le C_t y_t \quad t = 1,..,N \quad (4)$$

$$s_t \ge 0;\ x_t \ge 0;\ y_t = 0/1 \quad t = 1,..,N \quad (5)$$

The objective function (1) represents the total management costs, including the production (and/or purchasing), inventory and setup or ordering costs. Constraints (2) reproduce the demand satisfaction and inventory balance constraint for each period. Conditions (3) impose that inventory levels at the beginning and the end of the planning horizon are equal to zero. Constraints (4) allow a positive production (constrained between 0 and a value C_t) in period t if and only if the setup variable is equal to 1; in particular, the problem turns out to be uncapacitated for large values of C_t ($C_t \ge \sum_{k=t,N} d_k$, for every specific time period t). Constraints (5) express the non-negativity and binary restrictions on the variables. As known, model (1)–(5) has O(N) constraints in O(N) variables.

Wagner and Whitin (1958), in order to avoid trivial solutions to the problem, introduced a condition on the production and inventory costs, i.e. $h_t + p_t - p_{t+1} \ge 0$ (Wagner–Whitin cost condition). This condition assures that if setups occur in both periods t and $t+1$, it is more convenient to produce directly in period $t+1$, as there is no speculative reason for early production (Pochet and Wolsey, 1995). Zangwill (1966) further clarified that inventory costs h_t and setup/ordering costs f_t should be intended as non-negative.

Zangwill (1969) provided an interesting and fruitful interpretation of the problem as a fixed charge network problem. In Fig. 1, a network representation for a generic instance with N periods is provided. The flow on the generic arc $(0, t)$ represents the production in period t (x_t) while flow on arc (t, $t+1$) reproduces the stock at the end of period t (s_t). This way, constraints (2) can be interpreted as flow conservations conditions at each node t while constraints (4) indicate that, in presence of flow on arc $(0, t)$, this value cannot exceed the capacity of this arc. In practice the problem consists in defining the production inflows (x_t) able to satisfy the outflows (d_t) with the minimum cost, also using possible holdover flows accumulated in the previous periods (s_{t-1}).

This representation can also be used by reversing flows x_t and d_t as shown in Fig. 2. In this case, outflows (x_t) have to be determined in order to absorb the sum of the demand inflows (d_t) and of holdover flows from the previous period (s_{t-1}). Of course, in this case, in the formulation, constraints (2) have to be written reversing the signs of the variables x_t and parameters d_t, providing the following:

$$s_t = s_{t-1} - x_t + d_t \quad t = 1,..,N \quad (2')$$

In the case of a multi-item problem, introducing the index $j \in \{1..M\}$ representing one of the M items whose production has to be planned, and considering each parameter and variable indexed by both$j$ and t, the formulation of the multi-item DLSP, also known as Capacitated Lot-Sizing Problem (CLSP) becomes

$$\min z = \sum_{t=1}^{N} \sum_{j=1}^{M} (p_{tj} x_{tj} + h_{tj} s_{tj} + f_{tj} y_{tj}) \quad (6)$$

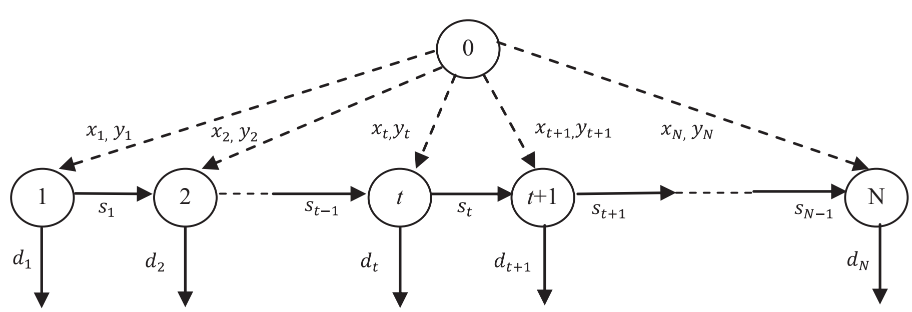

Fig. 1. Representation of a DLSP as network flow problem.

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