Int. J. Production Economics ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Lot-sizing policies for deteriorating items with expiration dates and partial trade credit to credit-risk customers

Jiang Wu [a], Ya-Lan Chan [b,*]

[a] School of Statistics, Southwestern University of Finance & Economics, Chengdu 611130, China
[b] Department of International Business, Asia University, Taichung, Taiwan 41354, ROC

ARTICLE INFO

Article history:
Received 29 December 2013
Accepted 31 March 2014

Keywords:
EOQ
Trade credit
Deteriorating item
Expiration date
Credit-risk customer

ABSTRACT

In practice, a credit-worthy retailer frequently receives a permissible delay on the entire purchase amount without collateral deposits from his/her supplier (i.e., an up-stream full trade credit). By contrast, a retailer usually requests his/her credit-risk customers to pay a fraction of the purchase amount at the time of placing an order, and then grants a permissible delay on the remaining balance (i.e., a down-stream partial trade credit). In addition, many products such as blood banks, pharmaceuticals, fruits, vegetables, volatile liquids, and others deteriorate constantly and have their expiration dates. However, not many researchers have taken the expiration date of a deteriorating item into consideration. The purpose of this paper is to establish optimal lot-sizing policies for a retailer who sells a deteriorating item to credit-risk customers by offering partial trade credit to reduce his/her risk. The proposed model is a generalized case of many previous models. By applying theorems in pseudo-convex fractional functions, we can easily prove that the optimal solution not only exists but is also unique. Moreover, we propose three discrimination terms, which can easily identify the optimal solution among all possible alternatives. Finally, some numerical examples are presented to highlight the theoretical results and managerial insights.

1. Introduction

Harris (1913) established the classical economic order quantity (EOQ) model based on the assumptions that the purchase items are non-perishable and can be sold indefinitely, and the retailer must pay for the entire purchase cost as soon as the purchase items are received. In reality, many products (e.g., fruits, vegetables, medicines, volatile liquids, blood banks and others) not only deteriorate continuously (e.g., evaporation, obsolescence, and spoilage) but also have their expiration dates. Ghare and Schrader (1963) derived a revised form of the economic order quantity (hereafter EOQ) model by assuming exponential decay. Then Covert and Philip (1973) extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Dave and Patel (1981) considered an EOQ model for deteriorating items with time-proportional demand when shortages were prohibited. Sachan (1984) further extended the model to allow for shortages. Goswami and Chaudhuri (1991) generalized an EOQ model for deteriorating items from a constant demand pattern to a linear trend in demand. Hariga (1996) established optimal EOQ models for deteriorating items with time-varying demand. Goyal and Giri (2001) provided a survey on the recent trends in modeling of deteriorating inventory. Skouri et al. (2009) considered inventory models with ramp-type demand rate and Weibull deterioration rate. Skouri et al. (2011) further generalized the model to add a permissible delay in payments under consideration. Dye (2013) provided some results on finding the optimal replenishment and preservation technology strategies for a non-instantaneous deteriorating inventory model. Recently, Chen et al. (2013b) proposed economic production quantity (EPQ) models for deteriorating items. All the above mentioned papers did not consider the fact that deteriorating items have their expiration dates. In fact, the study of deteriorating items with expiration dates has received a relatively little attention in the literature. Currently, Bakker et al. (2012) provided an excellent review of inventory systems with deterioration since 2001.

In practice, a seller frequently offers his/her buyers a permissible delay in payment (i.e., trade credit) for settling the purchase amount. Usually, there is no interest charge if the outstanding amount is paid within the permissible delay period. However, if the payment is not paid in full by the end of the permissible delay period, then interest is charged on the outstanding amount. Goyal (1985) proposed an EOQ model under conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's model for deteriorating items. Jamal et al. (1997) further generalized Aggarwal and Jaggi's model to allow for shortages. Chang et al. (2003) developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Huang (2003) proposed an EOQ model in which the supplier offers

* Corresponding author. Tel.: +886 4 2332 3456x48038.
E-mail address: yalan@asia.edu.tw (Y.-L. Chan).

http://dx.doi.org/10.1016/j.ijpe.2014.03.023
0925-5273/

the retailer a permissible delay and the retailer in turn provides his/her customers another permissible delay to stimulate demand. Ouyang et al. (2006) established an EOQ model for deteriorating items to allow for partial backlogging under trade credit financing. Liao (2007) presented an EPQ model for deteriorating items under permissible delay in payments. Teng (2009) developed ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers. Hu and Liu (2010) presented an EPQ model with permissible delay in payments and allowable shortages. Teng et al. (2011) extended an EOQ model for stock-dependent demand to supplier's trade credit with a progressive payment scheme. Teng et al. (2012) generalized traditional constant demand to non-decreasing demand. Lou and Wang (2013) proposed an integrated inventory model with trade credit financing in which the vendor decides his/her production lot size while the buyer determines his/her expenditure. Lately, Chen et al. (2013a) built up an EOQ model when conditionally permissible delay links to order quantity. Most of the above mentioned articles assumed that buyers are good-credit customers and receive full trade credit. Hence, the use of other trade credit strategies to reduce default risks with credit-risk customers has received a relatively little attention in the literature. Recently, Seifert et al. (2013) presented an excellent review of trade credit financing. Some relevantly recent articles in trade credit financing were developed by Chern et al. (2013), Taleizadeh (2014), Wu et al. (2014), and Yang et al. (2013).

In this paper, an EOQ model for deteriorating items with expiration dates was developed in a supply chain with up-stream full trade credit and down-stream partial trade credit financing. The proposed model is a generalized case of Goyal (1985), Teng (2002), Huang (2003), Teng and Goyal (2007), Teng (2009), and Chen and Teng (2014). Further, by applying the existing theoretical results in pseudo-convex fractional functions, we can easily prove that the optimal solution not only exists but also is unique. Moreover, we propose three discrimination terms to identify the optimal solution among possible alternatives. Finally, some numerical examples are presented to highlight the theoretical results and managerial insights.

2. Notation and assumptions

For simplicity, the notation and the assumptions used through the paper are presented below.

2.1. Notation

o	the retailer's ordering cost per order in dollars
c	the retailer's purchasing cost per unit in dollars
p	the market price per unit in dollars (with $p>c$)
h	the retailer's average stock holding cost per unit per year in dollars
I_c	the interest charged per dollar per year in stocks by the supplier
I_e	the interest earned or return on investment per dollar per year
t	the time in years
$I(t)$	the retailer's inventory level in units at time t
$\theta(t)$	the time-varying deterioration rate at time t, where $0 \le \theta(t) \le 1$
m	the maximum lifetime of the deteriorating item in years
S	the supplier's permissible delay period to the retailer in years
R	the retailer's permissible delay period to his/her customers in years
α	the fraction of the total purchase cost that a credit-risk customer must pay at the time of placing an order
D	the retailer's annual constant demand rate in units
T	the retailer's replenishment cycle time in years (decision variable)
Q	the retailer's economic order quantity in units
$TRC(T)$	the retailer's total relevant cost per year in dollars
T^*	the retailer's optimal replenishment cycle time in years
TRC^*	the retailer's optimal total relevant cost per year in dollars

2.2. Assumptions

Next, the following assumptions are made to establish the mathematical inventory model.

1. All deteriorating items have their expiration rates. The physical significance of the deterioration rate is the rate to be closed to 1 when time is approaching to the maximum lifetime m. To make the problem tractable, one can assume the same as in Wang et al. (2014) that the deterioration rate is

$$\theta(t)=\frac{1}{1+m-t}, \quad 0 \le t \le T \le m. \tag{1}$$

Note that it is obvious that the replenishment cycle time T is less than or equal to m, and Eq. (1) is a general case for non-deteriorating items, in which $m \to \infty$ and $\theta(t) \to 0$.
2. Replenishment rate is instantaneous and lead time is zero.
3. The retailer receives a full trade credit period of S years from his/her supplier (i.e., the retailer orders items at time 0, and must pay the supplier at time S without interest charges), and in turn provides a partial trade credit to his/her credit-risk customers who must pay α portion of the total purchasing cost at the time of placing an order as a collateral deposit, and then receive a permissible delay of R years on the outstanding amount (i.e., the customer orders items at time t, and must pay the delay payment at time $t+R$). Notice that to good-credit customers, the retailer may provides a full trade credit in which we simply set $\alpha=0$. Hence, the proposed model includes the special case in which the retailer offers a down-stream full trade credit to his/her customers.
4. If $S \ge R$, then the retailer deposits the sales revenue into an interest bearing account. If $S \ge T+R$ (i.e., the permissible delay period is longer than the time at which the retailer receives the last payment from his/her customers), then the retailer receives all revenue and pays off the entire purchase cost at the end of the permissible delay S. Otherwise (if $S<T+R$), the retailer pays the supplier the sum of money from all units sold by $S-R$ and the collateral deposit received from $t=0$ to S, keeps the profit for the use of the other activities, and starts paying for the interest charges on the items sold after $S-R$.
5. If $S \le R$, the retailer receives immediate payments from customers, and deposits the revenue into an interest bearing account until the end of the permissible delay S. As to delayed payment, the retailer must finance $(1-\alpha)cDT$ for delayed payment at $t=S$, and pay off the loan at $t=T+R$.

3. Mathematical formulation of the model

The retailer receives Q units at $t=0$. Hence, the inventory starts with Q units at $t=0$, and then gradually depletes to zero at $t=T$ due to the combination effect of demand and deterioration. Hence, the inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt}=-D-\frac{1}{1+m-t}I(t), \quad 0 \le t \le T, \tag{2}$$

Please cite this article as: Wu, J., Chan, Y.-L., Lot-sizing policies for deteriorating items with expiration dates and partial trade credit to credit-risk customers. International Journal of Production Economics (2014), http://dx.doi.org/10.1016/j.ijpe.2014.03.023