ARTICLE IN PRESS

Int. J. Production Economics ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Revisiting foundations in lot sizing—Connections between Harris, Crowther, Monahan, and Clark

Patrick Beullens

Mathematical Sciences and Southampton Business School and CORMSIS, University of Southampton, SO17 1BJ Southampton, United Kingdom

A R T I C L E   I N F O

Article history:
Received 12 October 2013
Accepted 11 April 2014

Keywords:
Deterministic inventory
Buyer–vendor coordination
Joint economic lot sizing
Quantity discounts
Multi-echelon
Pricing
Net present value
Stochastic regenerative demand

A B S T R A C T

While many review articles exist on (deterministic) lot sizing models used in the context of price and quantity discounts, buyer–vendor coordination, supply chain management, and joint economic lot sizing problems, they do not convey the impact of important findings which date back to at least 2002, or, in hindsight, to 1984. As a result, many recent articles still model the financial implications of lot sizing decisions without having the assurance that these models would help the firm(s) involved in maximising the Net Present Value (NPV). This paper therefore reviews these findings, while adding also its own contributions, as to convey the general importance to lot sizing theory. We show that the underlying principles used in the four key articles that have led to a division in modelling approaches are in fact all in line with NPV, and argue that therefore there should not be these discrepancies that currently persist in the literature. We establish the connections between these four strands of the literature using the solution to a simple variation of Harris' EOQ model, deriving thereby results from Boyaci and Gallego (2002) and Beullens and Janssens (2011), but showing their general applicability to any type of supply-chain structure. The breath of implications to deterministic lot sizing theory is illustrated using practical examples. We present a stochastic version of the model of Crowther (1964), which is arguably the least understood and applied model, but on the other hand the most important one in realising how these modelling strands can be unified.

1. Introduction

A theory of lot sizing is a set of models and algorithms to help find an optimal way of transferring batches of products through a supply chain in which these products undergo transformation, are held in stock, and are being moved on to the next stage(s). Such a theory would not be complete if it did not contain a set of founding principles that help explain in which context which models are applicable.

In particular, a firm may be interested in finding out how to model the impact that lot-sizing decisions will have on its future average profit. Since Harris (1913), it is well-known that lot sizing leads to financial holding costs that depend on the firm's opportunity cost of capital α. Key contributions to the theory that allows a more refined investigation were made in Hadley (1964), Grubbstrom (1967), Grubbstrom (1980), Porteus (1985), Haneveld and Teunter (1998), Van der Laan and Teunter (2002) and Boyaci and Gallego (2002). At the time of writing, more than a decade later, this should have generated an important shift in modelling the financial implications of lot-sizing, in particular in the context of price/quantity discounts, joint economic lot-sizing, and supply chain coordination. However, there are still numerous contributions being published which are demonstrably not in line with these results (see Section 9). No review article has done justice to the implications that should have followed; any discussion on this topic is surprisingly absent in recent review articles. This discrepancy justifies the publication of a review-type paper with a focus on explaining the generality of these principles and demonstrating their practical implications.

Contributions: Given the above discrepancy found in the current literature, this paper's first aim is to bring a cohesive and accessible report of the key findings from the above articles, and the few more recent relevant articles, as to convey an appreciation of the general impact on modelling the financial implications of lot-sizing. To this aim, we naturally 'reproduce' some of the results previously obtained, but also offer some new contributions. The origins of the four key strands of lot-sizing modelling approaches can be traced back to Harris (1913), Clark (1958), Crowther (1964), and Monahan (1984). Differences between the models of Crowther and Monahan, and implications for coordination, are discussed in Boyaci and Gallego (2002), and links to Harris and Clark are identified in Beullens and Janssens (2011). What this paper adds to this discussion is a deeper understanding: by reviewing the underlying reasoning of the authors of these classic papers, we see that there is in fact an important connection not previously

E-mail address: P.Beullens@soton.ac.uk

http://dx.doi.org/10.1016/j.ijpe.2014.04.010
0925-5273/

Please cite this article as: Beullens, P., Revisiting foundations in lot sizing—Connections between Harris, Crowther, Monahan, and Clark. International Journal of Production Economics (2014), http://dx.doi.org/10.1016/j.ijpe.2014.04.010

recognised. We prove how this could have led to an alternative history in which the current discrepancies in modelling approaches would not have existed, but would have been replaced by one unified approach, from as early as 1984.

In Harris (1913), when deriving the financial value of the average stock, Harris mentions that an *irregular demand pattern* would add additional complication, but concludes that it can generally be neglected or applied as a correction factor to the result. What Harris really meant with this comment is not known, but various plausible interpretations have been investigated (see Section 3). In this paper we will assign a different interpretation to Harris' comment about irregular demand patterns to arrive at a simple extension of his EOQ model. This model (in Section 3) is used to prove the connections among the four strands of lot sizing models investigated. Boyaci and Gallego (2002) were arguably first in deriving this result for the single supplier–multiple buyers setting, and Beullens and Janssens (2011) for the serial multi-echelon supply chain. Our contribution here, by viewing it as the result of an extension to Harris' model, is one of insight, namely, that the result represents the profit function of most firms more accurately in any supply-chain configuration. This is of pedagogical interest, since the generality of the key principle can be conveyed in a very natural manner in any introductory textbook on the topic of lot sizing.

As we will see, it is Crowther's model that is the key ingredient that is either missing or misinterpreted in the mainstream literature, while it nevertheless should appear naturally as part of the profit function of a firm. So far, (the relevance of) Crowther's model has only been shown to apply in deterministic models. In this paper, we generalise the profit function of the firm derived in Section 3 and derive a stochastic version of Crowther's model (Section 10). This proves its relevance beyond deterministic models, and the result also opens new avenues for further research.

To show the practical implications of this more unified theory to deterministic lot sizing, we present seven examples (Section 7) that complement those presented in earlier work. We illustrate why some classic inventory models will (1) underestimate the value of price discounts and do not find the optimal scheme; (2) underestimate the value a firm is willing to invest in reducing its unit purchase price; (3) underestimate the value of joint lotsizing and do not find the joint optimal solution; (4) miscalculate side-payments that firms should exchange in order to arrive at a Nash Equilibrium; and, (5) how these errors will propagate in a multi-echelon supply chain; we further show (6) the impact of payment structures, and (7) that the conditions under which the archetypal Joint Economic Lot Sizing (JELS) models can lead to equivalent results as those derived from NPV principles should necessarily deviate from the so-called 'conventional' payment assumptions (see further). In particular, we present two specific payment structures under which some JELS models from the literature do lead to results that are equivalent to NPV-derived JELS models. In general, we show that JELS research should pay explicit attention to the underlying payment structures if insights are to be derived concerning the relative benefits to the individual firms of adopting JELS solutions. This should be an important component of such research, but it is often missing.

Methodology: The main vehicle for our investigation is the use of Net Present Value (NPV) reference models. NPV modelling has been used at least for 50 years to check whether the lot sizing models we investigate are capable of maximising the NPV of the future profits of the firm(s) involved. The NPV value to the firm of its lot sizing activity is found as the Laplace transform of a cash-flow function (Grubbstrom, 1967) in which the Laplace frequency is taken to be the firm's opportunity cost of capital rate α. As in Hadley (1964) and Grubbstrom (1980), we construct from this the Annuity Stream (AS) function. The AS is the constant stream of payments having the same NPV as a given stream of payments. In line with the models under investigation, we consider a continuous time and infinite horizon situation, and hence AS $\equiv \alpha$NPV. The linear approximation of exponential terms in α (containing decision variables) results in a profit function that can be compared with the functions in the classic lot sizing models.

The validity of a linearised model in approximating the AS is limited to situations in which cycle times are not much larger than 1 year, when $\alpha \approx 0.2$ (time measured in years). This is an acceptable limitation in many applications. Throughout its presentation, this paper assumes that payments for set-ups and production or procurement occur in full whenever a batch arrives at the firm, unless otherwise stated. This is called a *conventional* payment structure in Beullens and Janssens (2014). In Section 7.6 we extend this to a class of other payment structures, and show that by using a suitable substitution of parameters, the functional forms of the terms obtained under conventional payment structures are preserved.

Despite the technique of NPV-based modelling being quite well-known, it is somewhat surprising that the model of Crowther (1964) was not investigated in this manner until Boyaci and Gallego (2002), and that these findings do not seem to have made much of an impact as of yet (see Section 9). This paper is therefore also a tribute to these authors, in the hope that future lot sizing theory will recognise the importance of their contributions. In Section 10, we transfer this key result to a stochastic setting by presenting, to our knowledge, the first NPV-derived stochastic version of the model of Crowther (1964).

The paper's further organisation follows an order based on logic rather than chronology. We therefore introduce the extension of Harris' model using NPV-principles after the discussion of Harris (1913), and discuss Clark (1958) after Crowther (1964) and Monahan (1984). We postpone a discussion on relevant literature (Sections 8 and 9), after having established the key insights. Since models from various sources are discussed and compared, we introduce our own set of notation in an aim to be consistent throughout, but we pay careful attention to reproduce the original results accurately.

2. Harris (1913)

In the EOQ problem (Harris, 1913), a firm is to satisfy without shortages a constant demand rate y per unit of time for an item, by procuring or producing at infinite rate in batches of equal size $Q = yT$ per unit of item, where T is the cycle time. The cost for acquiring each unit of item is c, and the generation of each batch results in a fixed set-up cost s. The cost for interest and depreciation on stock is α per monetary unit and unit of time. All other parameters given constants, the optimal lot-size Q that minimises the whole cost per unit of item and unit of time, and which is the sum of interest charges, set-up costs, and purchasing costs, as given by

$$\frac{\alpha}{2y}(cQ+s)+\frac{s}{Q}+c, \tag{1}$$

is found to be $Q^* = \sqrt{2sy/\alpha c}$. In addition to the derivation of this main result, Harris introduces the concepts of order lead-time and reorder point, and discusses their relationship; provides insight into the (in)sensitivity but asymmetry in the total costs around Q^*; and argues that the simplicity of the formula makes it easy to apply in practise.

Much of his reasoning remains convincing to date, and has been reconfirmed by extensive additional analysis. The EOQ model, for example, has been shown to be fairly accurately in line