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Lot sizing in case of defective items with investments to increase the speed of quality control

Zsuzsanna Hauck, József Vörös *

University of Pécs, Faculty of Business and Economics Pécs, Rákóczi 80, Hungary

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ABSTRACT

In many cases the quality of each item in a lot is checked. Speeding up the quality checking process increases the responsiveness of the system and saves cost. The percentage of defective items is a random variable and two models are proposed. In one of the models the system remains always at the same state, while in the other one after each order cycle, the state of the system may change, thus the percentage of defective items may be different in consecutive periods. In both cases the speed of the quality checking is a variable, and procedures are provided to find the optimal lot sizes and screening speed for general and specific investment cost functions. The characteristics of the two model settings will largely be different when the percentage of defective items is high. Among the important managerial insights gained is that a high unit backlogging cost, especially spurs the system to invest more intensively into improving the quality checking process.

1. Introduction

We consider an inventory system when products arrive in lots and the quality of each item in a lot is checked in order to decide whether it is acceptable or not. Defective items accumulating during the screening process are transported back to the supplier in a lot, reworked, or sold, and the good ones are used to satisfy demand. Demand is constant over time. The issue is interesting when we intend to determine the optimal ordering quantity, or when we plan the optimal production quantity. In producing medical instruments for example, when software is installed, the quality of hardware is always checked, i.e. the quality of each item is checked. The portion of defective items is considered to be a random variable. This basic concept was defined by Salameh and Jaber [15], who have inspired significant number of new papers, and this work directly belongs to this stream as well. Not long ago, Khan et al. [8] wrote a comprehensive summary on EOQ models including the problem of defective items, and this fact gives the flexibility of not summarizing the main results again but focusing on the relevant issues only. Salameh and Jaber [15] explicitly determined the optimum ordering quantity by taking the minimum of the expected value of the inventory and setup costs over unit time. Later, Maddah and Jaber [13] suggested a new process in which we have to minimize the ratio of the expected value of inventory and setup costs occurring in a cycle and the expected length of a cycle.

Voros [18] pointed out that different model settings can be aligned to each of these procedures. The original Salameh and Jaber [15] procedure gives the optimal lot size for the model when the system randomly gets into a state at the beginning, but the consecutive cycles inherit this state. On the other hand, when cycles are independent and may get into different states in each cycle, the Maddah and Jaber [13] procedure gives the optimum lot size.

Both approaches use an important assumption, namely that $p \leq 1-z$, where p is a random variable defined in [0, 1], denoting the fraction of the defective items in a lot, while z is a positive number, and $0<z<1$. Both models assume that the fraction of defective items is low enough to avoid shortages. In our case, when we assume that the speed of the quality checking process is a decision variable, this assumption will be easily violated and shortages will occur frequently. Papchristos and Konstantaras [14] and later Khan et al. [8] in their summary expressed that the condition to avoid shortages mentioned above is not sufficient to prevent non planned shortages. Papchristos and Konstantaras [14] also pointed out that even when p is replaced in the constraint by its expected value, the condition is still not sufficient to prevent non planned shortages. They expressed the view that there is no simple sufficient condition to prevent non planned shortages.

Let us note that shortages may easily occur due to machine break downs as well, and the production-inventory-maintenance literature handles the problem in many ways. Jonrinaldi and Zhang [7] for example, similarly to Berthaut et al. [1], protect the supply system by developing safety stocks (and consequently shortages

* Corresponding author.
E-mail addresses: hauckzs@ktk.pte.hu (Zs. Hauck), voros@ktk.pte.hu (J. Vörös).

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will occur), while in Wee and Widyadana [20] consider the cost of lost sale.

Voros [18] omitted the constraint intended to avoid shortages and proposed explicit solutions for both the independent and the connected cycle case. This work extends the work of Voros [18] assuming that the speed of the quality checking, the screening rate, is a decision variable, and it can be improved by investment, or by increasing the capacity of the quality checking process. The paper characterizes the relevant cost functions and proposes procedures to determine the optimal lot sizes and screening rates, and as consequence, the optimum investment to increase screening capacity.

As mentioned earlier, the concept of investment to increase the screening capacity, intensively requires the handling of backlogging cases however, these backlogging cases are not intentional. They are not planned, they will occur randomly. In the literature, probably one of the closest cases is presented by Khan et al. [9], where shortages result from low screening rates. By learning, backlogged demand will be satisfied, but the backlog is not developed by random events. The concept of learning turns back in one of their latest work, Khan et al. [10], where errors may occur in quality inspection and learning is incorporated into the model intended to determine the joint lot sizes in a supply chain. In these models after screening and finding defective items, supply batches are all accepted. For cases when the entire batch may be defective and therefore rejected on arrival, Skouri et al. [17] analyze a model and characterized the lot size. Liu and Yang [12] developed a model in which the process randomly produces defective items and the expected profit is maximized over the expected cycle time. They considered a system that resumes itself once a part of demand is not satisfied due to process quality problems. Jaber et al. [5] also maximize profit over time however, demand depends on both quality and price. Hsu and Yu [4] revisited the model of Salameh and Jaber [15] with the inclusion of one-time-only discount, and characterized the optimum lot sizes for different cases. With the inclusion of backordering, Wee et al. [19] extended the original inventory model concept with imperfect quality, but while backorders were satisfied, the quality was perfect. Taking into account that when satisfying backorders defective products may occur, Eroglu and Ozdemir [2] derived lot sizing formulas for models with defective items. Skouri et al. [16] develop a model with partial backlogging and Weibull deterioration rate for on-hand inventory and they consider the holding cost of deteriorated items as well. The system resumes itself by filling the inventory up to a certain level. Despite of these model developments, neither in the Khan et al. [8] summary, nor in other works, have we found any sign of discussing the problem of unplanned shortages with the possibility of improving the speed of screening. The next section identifies this model, and Sections 3.1 and 3.2 analyze the developed models for connecting and independent cycles, respectively. Section 4 gives conclusions.

2. Model identification

Let us consider the basic Salameh and Jaber [15] lot sizing problem with defective items when the arriving lot is fully screened and the defective items may be detected completely. A key characteristic of this model is the usage of a condition as a sufficient one for the fraction of defective items in a lot. Namely, the assumption is that $p \leq 1-z$, where p is a random variable defined in [0, 1], denoting the fraction of the defective items in a lot, while z is a positive number, and $0<z<1$. The lot is subjected to a 100% inspection, and in most cases z is the ratio of the demand to the screening rate, i.e. $z=D/x$, where D denotes the demand per day to be satisfied, and is constant over time, while x denotes the number of items checked per day. The assumption intends to avoid shortages, as the assumption can be expressed as $p \leq 1-D/x$, from which $x(1-p) \geq D$ follows. This says that when x units are inspected in a day, each day the number of perfect items may not be less than the daily demand. When the screening of a lot is over, which is at time Q/x - where Q is the lot size (also in units), defective items leave the system (sold, sent back to the supplier, or dismantled). Fig. 1, which depicts the situation, is well known in the literature.

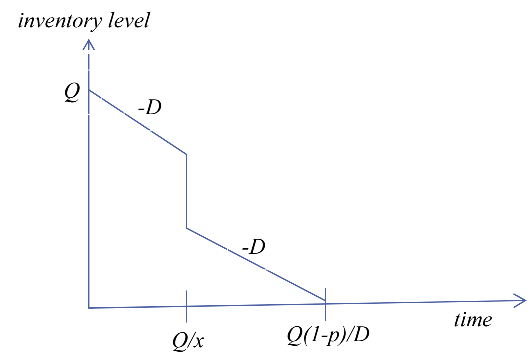

Fig. 1. The basic Salameh and Jaber [15] model.

When we assume that we can invest into the improvement of screening capability, not the lot size Q, but x will be a variable as well. Not violating the assumption $x(1-p) \geq D$ can hardly be expected, except when we assume that the current screening rate, denoted by x_0, is so large that the constraint $x_0(1-a) \geq D$ is valid, where a denotes the highest value of p with positive probability. Based on Fig. 1, when the assumption $x(1-p) \geq D$ is not violated, the holding cost per cycle (HCC1) can be expressed as

$$HCC1(Q,x) = h[\int_0^{Q/x}(Q-Dt)dt + \int_{Q/x}^{Q(1-p)/D}(Q(1-p)-Dt)dt]$$
$$= hQ^2(2pz+(1-p)^2)/2D, \quad x_{max} \geq x \geq x_0, \qquad (1)$$

where x_{max} denotes the maximum achievable screening capacity per day, and h is the daily holding cost per unit. Based on Fig. 1, let us note that besides given Q and D, the length of the cycle exclusively depends on the probability variable p and is independent of the rate of screening. When a lot contains non-defective items, the length of the cycle will be positive. However, the inventory level depends on the rate of screening as at time when screening is finished, defective items leave the system. If we are able to complete the screening task during a shorter time, inventory cost will decrease, thus it makes sense to consider a trade-off between the cost of inventory and investment to increase the screening speed.

On the other hand, when $x(1-p) \geq D$ is not true, shortages occur. Let us note again that even without the assumption that the screening rate were variable, Papchristos and Konstantaras [14] pointed out that even when p is replaced in the constraint $p \leq 1-z$ by its expected value, still the condition is not sufficient to prevent non-planned shortages. Thus when the screening rate is a variable, the condition $x(1-p) \geq D$ is violated, and shortages will occur.

In case of shortages, demand may not be satisfied partially or completely. We assume our product owns monopolistic features and other suppliers have no access to customers. Due to this fact we do not lose customers, so a backlog will develop, and as supposed by many basic handbooks (see for example [3]) backlog will continuously accumulate. Interestingly, in this case there will be on hand inventory and backlog simultaneously during the screening period. Until the time when screening is finished, the non-screened units (defective and perfect ones) are on inventory, but at the same

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