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Joint redundancy and imperfect preventive maintenance optimization for series–parallel multi-state degraded systems

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ABSTRACT

This paper formulates a joint redundancy and imperfect preventive maintenance planning optimization model for series–parallel multi-state degraded systems. Non identical multi-state components can be used in parallel to improve the system availability by providing redundancy in subsystems. Multiple component choices are available in the market for each subsystem. The status of each component is considered to degrade with use. The objective is to determine jointly the maximal-availability series–parallel system structure and the appropriate preventive maintenance actions, subject to a budget constraint. System availability is defined as the ability to satisfy consumer demand that is represented as a piecewise cumulative load curve. A procedure is used, based on Markov processes and universal moment generating function, to evaluate the multi-state system availability and the cost function. A heuristic approach is also proposed to solve the formulated problem. This heuristic is based on a combination of space partitioning, genetic algorithms (GA) and tabu search (TS). After dividing the search space into a set of disjoint subsets, this approach uses GA to select the subspaces, and applies TS to each selected sub-space.

1. Introduction

To improve the performance of a multi-state degraded system, preventive maintenance (PM) plays a key role [1]. Perfect PM is aimed at making the MSS 'as good as new', while imperfect PM may bring the MSS back to an intermediate state between the current state and the perfect functioning state. In practice, both redundancy and maintenance are used to provide a required level of system reliability. Engineers typically try to achieve this level with minimal cost by solving the problems of redundancy optimization or maintenance optimization separately. However, a trade-off exists between investments into system redundancy and its maintenance cost. As it is explicitly pointed out in [33], the optimal design should consider both of these factors in order to reach a solution that provides the desired system performance at minimum cost.

On one hand, the redundancy allocation problem (RAP) involves selection of components and levels of redundancy to maximize system performance. The RAP is NP-hard [2]. It has attracted considerable attention from the research community. The great majority of the existing papers on the RAP use traditional binary-state reliability. It is assumed in binary-state reliability modeling that a system and its components may experience only two possible states: good and failed. The RAP for binary-state series–parallel systems has been studied in many different forms, and by considering numerous approaches and techniques. It has been solved by using optimization approaches and techniques such as dynamic programming, integer programming, mixed-integer non-linear programming, heuristics and meta-heuristics: see for example [3–5] for an extensive overview of these techniques. Nevertheless, in many real-life cases, this binary-state assumption may not be adequate. In multi-state reliability modeling, the system may rather have more than two levels of performance ranging from perfect functioning to complete failure. A multi-state system (MSS) may perform its task at different intermediate states between working perfectly and total failure. The presence of degradation is a common situation in which a system should be considered to be a MSS. Degradation can be caused by system deterioration or by variable ambient conditions. In this case, the failure rate depends on the status of the system

Abbreviations (the singular and plural of an acronym are always spelled the same): GA, genetic algorithms; TS, tabu search; PM, preventive maintenance; RAP, redundancy allocation problem; MSS, multi-state system; UMGF, universal moment generating function; SP, space partitioning

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Nomenclature

Δ	number of series MSS subsystems
δ	index for subsystem, $\delta=1, 2,\ldots,\Delta$
v_δ	number of available components choices for subsystem δ
φ	index version for component
$\xi_{\delta\varphi}$	number of components of type φ used in subsystem δ
$\boldsymbol{\xi}_\delta$	vector $(\xi_{\delta 1}, \xi_{\delta 1}, \ldots, \xi_{\delta v_\delta})$
$\boldsymbol{\Xi}$	a string which defines the entire system structure, $\boldsymbol{\Xi}=(\xi_{11}, \xi_{12}, \ldots, \xi_{1v_1}, \xi_{21}, \xi_{22}, \ldots, \xi_{2v_2}, \ldots, \xi_{\Delta 1}, \xi_{\Delta 2}, \ldots, \xi_{\Delta v_\Delta})$
κ	index indicating the type of preventive maintenance
$\psi_{\delta\varphi\kappa}$	binary decision variable that is equal to 1 if a preventive maintenance of index κ is performed on components of type φ used in subsystem δ
$\boldsymbol{\vartheta}$	vector of $\psi_{\delta\varphi\kappa}$
$N_{\delta\varphi}$	maximum number of components of version φ in parallel belonging to subsystem δ
$C_{\delta\varphi}$	procurement cost of a component of version φ in subsystem δ
$CR_{\delta\varphi}$	repair cost incurred per unit time for a component of version φ in subsystem δ
$CPM_{\delta\varphi}$	preventive maintenance cost incurred per unit time for a component of version φ in subsystem δ
$CM(\Xi, \vartheta)$	total maintenance cost
$C(\boldsymbol{\Xi})$	total procurement cost
$TC(\boldsymbol{\Xi}, \boldsymbol{\vartheta})$	expected total cost
F	number of partitioned intervals
f	index for partitioned intervals
T	system life cycle (lifetime expectation)
T_f	a partitioned interval in T
W_f^0	required MSS performance level for T_f
$\mathbf{W}^0$	vector of W_f^0
W_s	total capacity of the system
$W(t)$	output performance level of the MSS at time t
$A(\boldsymbol{\Xi}, \boldsymbol{\vartheta}, \mathbf{W}^0)$	stationary availability index of the overall multi-state series–parallel system
B_0	a pre-specified maximum budget
mnli	maximum number of local iterations without improvement
N_s	number of randomly-constructed solutions in the initial population of GA
N_c	number of genetic cycles
N_{rep}	number of reproduction-selection procedures per genetic cycle
α	amplification parameter in the penalized objective function

which can degrade gradually. The reliability analysis of such degraded systems should consider multiple operational states to take into account multiple degradation levels [46]. The basic concepts of MSS reliability were first introduced in [6–9]. These works defined the system structure function and its properties. They also introduced the notions of minimal cut set and minimal path set in MSS context, and studied the notions of coherence and component relevancy. A literature review on MSS reliability can be found for example in Ref. [10]. The methods currently used for MSS reliability estimation are generally based on four different approaches: (*i*) the structure function approach which extends Boolean models to the multi-valued case (e.g., [7–9]); (*ii*) the Monte-Carlo simulation technique (e.g., [11]); (*iii*) the Markov process approach (e.g., [12,13]); and (*iv*) the universal moment generating function (UMGF) method (e.g., [14,15]). In Ref. [50], the author presents a method that combines the Markov process approach with the UMGF. These approaches are often used by practitioners, for example in the field of power systems reliability analysis [10,16]. In practice, different reliability measures can be considered for MSS evaluation and design [17,18]. For example, the availability of a repairable MSS is defined by the system ability to meet a customer's demand (required performance level). In power systems, it is the ability to provide an adequate supply of electrical energy [16]. The RAP for series–parallel MSS is more recent than that of binary-state systems, and it has been much less studied in the literature. It was first introduced in [19], where the UMGF method [14] was used for the reliability calculation. Following these works, genetic algorithms (GA) were used for the homogeneous RAP of series–parallel MSS in [20,21], and extended to the non-homogeneous version of this problem in [22]. Other existing solution methods for the homogeneous case include an ant colony optimization in [23], heuristic algorithms in [24,25], and a tabu search (TS) approach in [26]. Also in [27,28], TS is combined with GA to solve reliability design problems.

On the other hand, for systems containing components with failure rates increasing in time, PM of the components can also be used to enhance system performance [47]. In Ref. [29], the authors study a deteriorating repairable MSS with an imperfect PM policy which is based on the failure number of the system. In Ref. [30], a model of MSS with state-dependent cost is considered. The state space of the system is partitioned into two subsets: the first represents all states of normal operations, while the second represents the single failure state. A periodic maintenance model is developed and the optimal cycle time of maintenance actions is determined over a specific finite horizon. More recently, in Ref. [31] the author develops a monotone process maintenance model for a MSS. A replacement policy which is based on the failure number of the system is studied. An analytical approach is used to determine the optimal replacement policy. In Ref. [32], the authors investigated the optimal replacement strategy for MSS incorporating imperfect maintenance quality. In Ref. [49], the authors deal with preventive maintenance optimization problem for multi-state systems (MSS). This problem consists of finding an optimal sequence of maintenance actions which minimizes maintenance cost while providing the desired system reliability level.

To our knowledge, the only existing papers dealing with the integration of redundancy allocation and PM planning are [33] and [48]. In [33], the authors presented an algorithm that can be used to determine both the optimal configuration for a multistate series–parallel system and the optimal schedule of cyclic replacements of system components. The main difference between Ref. [33] and the present contribution is that we rather consider imperfect PM (instead of preventive replacements by new components). Furthermore, we consider a model that maximizes the system availability under budget constraint, while the model studied in [33] corresponds to a cost minimization problem. In [48], the authors consider a joint reliability/redundancy optimization problem, where non-homogeneous multi-state system structure is optimized jointly with choosing technical actions affecting the transition rates of system components. Our redundancy/PM optimization problem can be seen as a special case of the general problem studied in [48]. However, the redundancy/PM