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EOQ and EPQ with linear and fixed backorder costs: Two cases identified and models analyzed without calculus

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Abstract

The EOQ and EPQ models with linear and fixed backorder costs are newly analyzed and all formulae are proven using only algebra. Two cases are identified, where backorders should and should not be allowed. The results are shown to include several known special cases.
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1. Introduction

Two recent papers in this journal, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) used an algebraic approach to prove the formulae for the EOQ and economic production quantity (EPQ) with a single cost of backordering, only linear (time dependent). Here we extend the approach to the more general models of EOQ and EPQ with two backorder costs, a linear and a fixed cost per unit. The only treatment of these models we could find was in Johnson and Montgomery (1974, pp. 26–33). In their analysis they used calculus and solved the system of equations resulting from the first-order conditions. They did not explicitly identify the two distinct cases we examine here. Our analysis is based entirely on an algebraic approach. In addition to the fact that it is of interest to demonstrate how a relatively complex model can be fully analyzed without derivatives, we obtain the explicit identification of the two cases, a result that does not appear as easy to do by using calculus.

First we examine the EOQ model and at the end present the results for the EPQ.

2. The EOQ model with two backorder costs

Notation:

Q = size of order

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S = size of backorders, $0 \leqslant S \leqslant Q$
TC = total cost per unit of time
D = demand per unit of time
K = cost of ordering
h = holding cost per unit, per unit of time
p = backorder cost per unit, per unit of time (linear backorder cost)
π = backorder cost per unit (fixed backorder cost)

The total cost is

$$\mathrm{TC} = \frac{KD}{Q} + \frac{h(Q-S)^2}{2Q} + \frac{pS^2}{2Q} + \frac{\pi DS}{Q}. \qquad (1)$$

In some presentations the total material cost cD, where c is the cost per unit, is included in the total cost. Since that constant term does not play any role here it is omitted. All terms in (1) are nonnegative, but by rewriting it is possible to focus on one term that may be positive or negative, and that leads to an important conclusion. The algebraic manipulation involves first combining the first two terms and completing a perfect square, using

$$2KD + h(Q-S)^2 = 2\sqrt{2KDh}(Q-S) + \left(\sqrt{h}(Q-S) - \sqrt{2KD}\right)^2.$$

After a couple of steps the total cost is rewritten in the following form:

$$\mathrm{TC} = \sqrt{2KDh} + \left(\pi D - \sqrt{2KDh}\right)\frac{S}{Q} + \frac{h}{2Q}\left((Q-S) - \sqrt{2KD/h}\right)^2 + \frac{pS^2}{2Q}. \qquad (2)$$

In this form, an important dichotomy may be observed, based on the term $(\pi D - \sqrt{2KDh})$. Two distinct cases arise, depending on the sign of this term, and they are discussed below. Before proceeding to the analysis of each of the two cases (*Case 1*: $\pi D \geqslant \sqrt{2KDh}$, and *Case 2*: $\pi D < \sqrt{2KDh}$), it is interesting to note that these two values have explicit meanings. $\sqrt{2KDh}$ is the cost per unit of time in the simple EOQ with no backorders, and πD is the total fixed backordering cost per unit of time if the entire demand is backordered. Thus, the comparison is between the per unit operating cost without backorders and the total fixed cost of backordering everything. If the latter exceeds the former, we will conclude, in Case 1, that it is too expensive to have any backorders. Conversely, in Case 2, we will show that backorders should be carried. In this dichotomy the size of the linear backorder cost (p) plays no role. As far as the parameter is concerned, p is never large enough, if finite, to make backordering too expensive. The distinction between what we refer to as "backorders too extensive" or "backorders attractive" is based entirely on the magnitude of the fixed cost π (relative to $\sqrt{2Kh/D}$).

Case 1: *Backorders too expensive*: $\pi D \geqslant \sqrt{2KDh}$. In this case, a simple examination of (2) is sufficient. All terms are nonnegative, and any non-zero choice of S would make the TC strictly greater than $\sqrt{2KDh}$. Reducing all the terms after the constant to zero minimizes the total cost, i.e., $S^* = 0$, $Q^* = \sqrt{2KD/h}$, and $\mathrm{TC}^* = \sqrt{2KDh}$. This demonstrates explicitly that in this case we get the basic EOQ without backorders. So far we have established that the optimal is $S = 0$ if $\pi D \geqslant \sqrt{2KDh}$. Actually, this statement can be generalized to an "if and only if" condition after we conclude that in Case 2, the optimal value of S is strictly positive.

Case 2: *Backorders attractive*: $\pi D < \sqrt{2KDh}$. Now the TC expression in (2) includes a negative term. The interesting question, from the algebraic point of view, is whether or not this term can be made sufficiently negative to outweigh the last two nonnegative terms. It turns out this is indeed always true. Using (2) it is difficult to make further progress directly, but we can return to (1) and obtain a different expression for the TC. In order to clarify the logic of the analysis, we point out what is the ultimate goal of the algebraic manipulations that follow. We attempt to rewrite the TC, if possible, in the following form:

$$\mathrm{TC} = a_0 + a_1(Q-A)^2 + a_2(S-B)^2. \qquad (3)$$

If it can be established that this can be done, if the coefficients are all nonnegative, and if A and B are valid values for Q and S, then we can reach an immediate conclusion. The total cost, from (1) can be rewritten as follows:

$$\mathrm{TC} = \frac{KD}{Q} + \frac{hQ}{2} + \frac{(h+p)S^2}{2Q} + \frac{(\pi D - hQ)S}{Q}$$

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