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Backorder penalty cost coefficient "*b*": What could it be?

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ABSTRACT

The classical *economic order quantity* (EOQ) model with planned *penalized backorders* (PB) relies on postulating a value for the backorder penalty cost coefficient, *b*, which is supposed to reflect the intangible adverse effect of the future loss of customer goodwill following a stockout. Recognizing that the effect of the future loss of customer goodwill should be not a direct penalty cost but a change in future demand, Schwartz [1966. A new approach to stockout penalties. Management Science 12(12), B538–B544] modified the classical EOQ-PB model by eliminating the backorder penalty cost term from the objective function and assuming that the long-run demand rate is a decreasing, strictly convex function of the customer's "disappointment factor" (defined as the complement of the demand fill rate) following a stockout, which in turn is an increasing, strictly convex function of the demand fill rate. He called the new model a *perturbed demand* (PD) model. Schwartz provided convincing justification for his PD model and presented several variations of it in a follow-up paper, but he did not solve any of these models. In this paper, we solve Schwartz's original PD model and its variations, and we discuss the implications of their solutions, thus filling a gap in the literature left by Schwartz. Moreover, having been convinced that Schwartz's approach is more valid than the classical approach for representing the effect of the loss of customer goodwill following a stockout, but also recognizing that the classical approach is far more popular than the PD approach, because of its simplicity and because of tradition, we use the solution of the PD model to infer the value of *b* in the classical model, thus providing one possible answer to the question, what could *b* be? A noteworthy implication of the solution of Schwartz's original PD model is that the optimal fill rate is always 0 or 1, rendering the inferred value of *b* in the classical model 0 or ∞, respectively. Suspecting that the property of the PD function which is most likely responsible for producing this "bang-bang" type of result is strict convexity, we show that for the case where the PD function is proportional to an integer power, say *n*, of the fill rate, the optimal fill rate is always 0 or 1, if and only if $n>1$, in which case the PD function is strictly convex in the fill rate.

1. Introduction

Anyone who has taken or taught a course in inventory management is likely to have pondered at how to quantify the cost incurred by a stockout. A stockout may incur an immediate, direct cost to the firm, as well as a future, indirect cost. The direct cost depends on whether the unfilled demand is backordered and eventually fulfilled with a delay, or is cancelled. In the first case, the direct cost is related to the delayed delivery and may include extra administration costs, material handling and transportation costs for expediting the backordered items, fixed or variable contractual penalties, the loss of profit from

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selling the backordered items at a discounted price, the interest on the profit tied up in the backorder, etc. In the second case, the direct cost is the lost profit of the cancelled demand. In many practical situations, part of an unfilled demand is backordered and part of it is cancelled. In most cases, the direct cost may be calculated with some effort. The indirect cost is much harder to evaluate. It is related to the loss of customer goodwill due to the stockout, which may lead to a temporary or permanent decline in future demand and market share, especially in a competitive market environment.

The quantification of the indirect cost of stockouts has long been an unsatisfactorily resolved issue in the literature. The difficulty in determining an appropriate penalty rate for the indirect cost of stockouts has prompted many researchers to replace this rate by a constraint on the customer service level. For example, Çentikaya and Parlar (1998) take this approach for the *economic order quantity* (EOQ) model with planned backorders which is at the center of our study in this paper too. This approach may seem more appealing to practitioners, but it only transposes the problem of estimating an appropriate penalty rate for stockouts to one of determining an appropriate customer service level.

The EOQ model with planned backorders is one of the earliest models in inventory theory that deals with stockouts. It relies on postulating a value for the backorder penalty cost coefficient, denoted by b, which is supposed to reflect the intangible adverse effect of the loss of customer goodwill following a stockout. We refer to this model as the *penalized backorders* (henceforth, PB) model. The PB model is based on the following assumptions. A firm buys a single type of items from a supplier, holds them in inventory, and sells them to its customers upon demand. The demand for items, denoted by D, is continuous and constant over time, procurement and delivery of the items are instantaneous, and unfilled demand is backordered. Finally, the gross profit (selling price minus purchase price) per item sold, denoted by p, the fixed order cost, denoted by k, the inventory holding cost per item per unit time, denoted by h, and the backorder penalty cost per item per unit time, denoted by b, are known and constant over time. The decision variables are the order quantity, denoted by Q, and the fraction of demand that is met from stock, known as *fill rate*, denoted by F.

All the parameters of the PB model, except b, may be more or less specified. Schwartz (1966) was one of the first to note that the effect of the loss of goodwill should not be a direct penalty cost of the type considered in the PB model, because the effect of goodwill loss is incurred not at the time of the stockout incident, but at a later time, due to the customer's disappointment caused by the stockout and his subsequent decision to lower his future demand. With this in mind, Schwartz (1966) modified the PB model by eliminating the explicit backorder penalty cost term from the objective function and assuming that the long-run demand rate—and hence the long-run average reward of the firm—is a function of the customer's "disappointment factor", which he defined as the fraction of demand not met from stock. Schwartz called the resulting model a *perturbed demand* (henceforth, PD) model.

To derive an analytical form of the perturbed demand as a function of the disappointment factor, Schwartz (1966) assumed the following customer response to stockouts. When a customer places an order and finds out that it cannot be delivered, he changes his a priori ordering pattern in the future by reducing the amount he would otherwise have bought in each of a number of future periods. The total amount that the customer does not buy because of the disappointment, denoted by B, and the maximum potential demand rate in a cycle with no disappointments, denoted by A, are finite. The above assumed customer response led to the following strictly convex long-run PD function:

$$D'(F') = \frac{A}{1+(1-F')B}, \tag{1}$$

where F' is the long-run average fill rate, and hence $1-F'$ is the fraction of demand not met from stock, i.e., Schwartz's disappointment factor. We note that throughout this paper, we shall be using the notation X' for variables and functions in the PD model whose equivalent variable/function in the PB model is denoted by X, to distinguish between the two models.

The PD model proposed by Schwartz (1966) replaces the indeterminable task of subjectively choosing b in the PB model with the better defined task of estimating parameters A and B of the PD function, $D'(F')$. Schwartz (1966) proposed a procedure for measuring parameters A and B from observed demand data. This procedure is based on the assumption that when a customer faces a stockout, he reduces the size of his next order by some amount, the following one by a smaller amount, the next by a still smaller, and so on, so that as time passes, he tends to forget about the disappointment; therefore, his subsequent orders will approach their original level, A.

Schwartz (1966) provided convincing justification for his PD model, but he did not solve it. In a follow-up paper, Schwartz (1970) continued his investigation of the PD model by formulating three different variations of it in which he replaced the explicit fixed order cost with a constraint on the order quantity, the interorder time, and the starting inventory in each cycle, respectively. For each variation he considered both cases with backlogging and lost sales. In all variations, he merely stated in a few lines the first-order condition for the optimal quantity of unfilled demand, but in none of these variations did he solve this condition or provide any further analysis, discussion, or insight. In this paper, we solve exactly the original PD model introduced by Schwartz (1966) and its three variations considered in Schwartz (1970), in the case of backlogging, thus filling a gap in the literature left by Schwartz. Moreover, we discuss the implications of the solutions.

In the last sentence of his conclusions, Schwartz (1970) wrote, "The Perturbed Demand approach to goodwill stockout penalties is both substantially more valid and more practical than any previously considered in the literature of inventory theory". We agree with the position that the PD approach to goodwill stockout penalties is in