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A newsboy problem with an emergency order under a general backorder rate function

Valentín Pando [a], Luis A. San-José [b,*], Juan García-Laguna [a], Joaquín Sicilia [c]

[a] Departamento de Estadística e Investigación Operativa, Universidad de Valladolid, Valladolid, Spain
[b] Departamento de Matemática Aplicada, Universidad de Valladolid, Escuela Técnica Superior de Ingeniería Informática, Paseo de Belén 15, 47011-Valladolid, Spain
[c] Departamento de Estadística, Investigación Operativa y Computación, Universidad de La Laguna, Tenerife, Spain

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ABSTRACT

In this paper, a generalization is presented of the newsboy problem where an emergency lot can be ordered to provide for a certain fraction of shortage. This fraction is described by a general backorder rate function which is non-increasing with respect to the unsatisfied demand. An exponential distribution for the demand during the selling season is assumed. An expression is obtained in a closed form for the optimal lot size and the maximum expected profit. A general sensitivity analysis of the optimal policy with respect to the backorder rate function and the parameters of the inventory system is developed. When the backorder rate function is described by some particular functions, its behavior is analyzed with respect to changes in the parameters. To illustrate the theoretical results, some numerical examples are also given.

1. Introduction

The newsboy problem is probably the most studied stochastic inventory model in inventory control theory and the one with most extensions in recent years. This problem reflects many real life situations and is often used to aid decision making in both manufacturing and retailing [14]. It is particularly important for items with significant demand uncertainty and large over-stocking and under-stocking costs. Since the review of the newsboy problem provided by [20], many researchers have paid considerable attention to this stochastic inventory problem. Thus, in the last few years, new papers on this topic have appeared in the literature.

In the newsboy problem, two main issues that give rise to possible extensions are considered: the potential use of surplus products at the end of the season and the possible existence of an emergency order to fill a shortage during the selling season. For example, the first approach is considered by [21], using rebates to sell the excess inventory below the regular price. In line with the second approach are, for instance, the papers by [19,22–24].

When the possibility of an emergency order exists, it is necessary to consider the behavior of customers faced with a shortage in the inventory. For instance, Gallego and Moon [14] supposed that all of them will be willing to wait for the emergency order, while [19] considered that only a fixed fraction of them will be willing to wait. However, what usually happens in practice is that customers' responses toward shortage depend on the backorder lead time through the magnitude of the shortage. Therefore, the fraction of customers for the emergency order is a mathematical function of the unsatisfied demand left by the initial order. Thus, Lodree [24] considered that this fraction is a decreasing linear function ranging from the value one (when there is no shortage) to the value zero (when the shortage attains a certain threshold from which the customer is no longer willing to wait for the emergency order). Lee and Lodre [22] has recently introduced two new functions to model different behaviors of customers: a negative exponential function for impatient customers and a cosinusoidal function for patient customers. Also in this line, Brito and de Almeida [7] has proposed a multi-attribute expected-utility model for the newsboy problem including in the objective function the possible impact of the unsold products on the environment and the service level impact on corporate image and customer' goodwill.

In the context of inventory models with continuous review, Padmanabhan and Vrat [26,27] also considered the assumption that the fraction of demand which is served with delay depends on the extent of outstanding orders to satisfy (i.e. on the net inventory level). This assumption on the rate of cumulative demand has been used by [9,11,25,33]. An alternative approach to model this situation consists in considering that the backlogged demand rate is variable and depends on the waiting time for the next replenishment. Abad [1] was the first work proposing two functions of the waiting time to model a customer's patient behavior. Subsequently, the papers by [2,3,6,8,12,28–30] have also followed this approach.

* Corresponding author. Tel.: +34 983423000x5707.
E-mail addresses: vpando@eio.uva.es (Valentín Pando), augusto@mat.uva.es (Luis A. San-José), laguna@eio.uva.es (Juan García-Laguna), jsicilia@ull.es (Joaquín Sicilia).

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Another important question in the newsboy problem is the probability distribution of demand. In the basic case, the general solution can be obtained for any probability distribution by using its cumulative distribution function. Dominey and Hill [10] assumed that the number of customer arrivals during the period follows a Poisson distribution and the sizes of individual customer orders are independent random variables with normal, gamma or lognormal distribution. Gallego et al. [13] analyzed the variation of the solution for normal, lognormal, gamma, negative binomial and Pareto distributions. For the problem with an emergency order, the exponential distribution is probably the most used for demand, sometimes together with uniform and normal probability distributions. So, Abdel-Malek et al. [4] considered the multi-product newsboy problem with exponential, uniform and beta distributions. Lodree [24], Khouja and Vergara [21] and Lee and Lodre [22] have analyzed inventory models in this same research line. Recently, another extensions for the newsboy problem have been considered by [5,16,18,32] or [17].

The purpose of this paper is to study the newsboy problem with an exponential distribution of demand and an emergency order. We model the backlogged fraction of the shortage as a general non-increasing function of the magnitude of the shortage defined over the interval [0,1]. This function represents the fraction of customers served with the emergency order. By using different functions, distinct situations can be modeled for the patience or impatience of customers faced with shortages of the inventory. This approach generalizes the models studied by other authors for the newsboy problem with emergency order and demand exponentially distributed. While in some works it is assumed that the backorder rate requires certain constraints to be satisfied, here the imposed assumptions are relaxed. Thus, in the model of [22], this function must be strictly decreasing, differentiable at least once and takes values from one down to zero. However, in this paper, we only suppose that the backorder rate function is a non-increasing function. Closed-form expressions are obtained for the optimal policy and the maximum expected profit, independent of the backorder rate function used in the model. As special cases of the model introduced here, we have the newsboy model studied by [24], the newsboy models with fixed backorder rate [19], completely backlogged shortages [14] and the basic newsboy model (see, for instance, [15]).

The paper is organized as follows. In Section 2, the problem is defined and the assumptions and notation to be used in the formulation of the model are established. In Section 3, the model is studied, obtaining the general solution for any backorder rate function. Also, a sensitivity analysis of the optimal solution and the maximum expected profit with respect to the backorder rate function and the parameters of the system is presented. In Section 4, the results previously shown are analyzed when the behavior of customers is described by some interesting particular functions. Moreover, an example of each case is provided to illustrate the theoretical results. Finally, some conclusions are given in Section 5.

2. Problem definition

In this paper, we consider a newsboy problem in which an emergency order to satisfy a certain fraction of the shortage during the selling season is allowed. Thus, if the demand x is greater than the stock size Q and $y=x-Q$ denotes the shortage, we suppose that the vendor has the possibility of supplying with delay a certain fraction $\beta(y)$ of the unsatisfied demand through an emergency order. This fraction $\beta(y)$ can be any non-increasing function (not necessarily continuous) of the quantity of shortage. Moreover, we suppose that the demand is described by a random variable X with an exponential distribution and we focus on the determination of the optimal order size Q^* which gives the vendor the maximum expected profit at the end of the selling period.

In what follows, we denote by v the unit selling price, and by c the unit purchasing cost. Furthermore, we consider that surplus items at the end of the season cause a unit effective holding cost c_H which can be negative (that is, it represents a revenue) when the possibility of selling them at a bargain price smaller than the unit purchasing cost exists, in line with the model considered by [21]. Hence, we assume that the total unit overstocking cost is $h=c_H+c>0$. Also, we assume that the unit purchasing cost for the emergency order c_B is greater than the initial unit purchasing cost c. That is, we suppose $c_B>c$ because emergency orders involve higher administrative costs than the initial order and also the manufacturing cost increases due to a lack of production planning and the need to serve these products with a lower lead time than the initial order. Thus, the difference between these two costs, $\omega=c_B-c>0$ denotes the unit extra cost of the emergency order. Furthermore, we suppose that each unmet demand causes a unit goodwill cost c_G, in addition to the unit cost for loss of profit $v-c$ and, therefore, the total unit cost of lost sales is $p=c_G+v-c>0$. Moreover, we consider that the unit selling price v is greater than the unit purchasing cost for the emergency order c_B; otherwise, the vendor would prefer to lose the sale rather than recover it by the emergency order. Thus, it results that the total unit cost of lost sales p is greater than the unit extra cost of the emergency order ω. Similar assumption is also made by [19,22,24].

Finally, we consider a new parameter of the inventory system, called the *emergency intensity*, defined by $\beta_o=\lim_{y\to 0^+}\beta(y)$. It represents the initial ratio of unsatisfied demand which is backlogged through the emergency order and, therefore, $1-\beta_o$ is the fraction of customers that are lost simply because there is a shortage in the inventory, independently of how small it may be. Note that if $\beta_o=1$, we revert to the assumption considered by [24,22].

The following table summarizes the notation used in this paper:

Q	order quantity, the *decision variable* (>0)
X	continuous random variable, which describes the demand during the selling season
x	observed value of demand (≥ 0)
$f(x)$	density function of X
$F(x)$	distribution function of X
μ	expected demand (>0)
y	shortage before the emergency order ($y=(x-Q)^+\geq 0$, where $(x-Q)^+=\max\{x-Q,0\}$)
$\beta(y)$	backorder rate function ($0\leq\beta(y)\leq 1$)
β_o	emergency intensity ($0\leq\beta_o\leq 1$)
c	unit purchasing cost before the selling season (>0)
v	unit selling price ($>c$)
c_H	unit effective holding cost for surplus items ($>-c$)
c_B	unit purchasing cost for emergency order ($<v$)
c_G	unit goodwill cost for lost sale (≥ 0)
Auxiliary parameters	
h	unit total overstocking cost ($h=c_H+c>0$)
ω	unit extra cost of the emergency order ($\omega=c_B-c>0$)
p	unit cost of lost sale ($p=c_G+v-c>\omega$)

Taking into account the previous assumptions and notation, if Q is the order size and x is the value for the demand, we have

Ordinary sales income $=v\min(Q,x)=vx-v(x-Q)^+$.
Income from sales of backlogged demand $=v(x-Q)^+\beta(x-Q)^+$.