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Trade-off between sample size and accuracy: Case of measurements under interval uncertainty

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ABSTRACT

In many practical situations, we are not satisfied with the accuracy of the existing measurements. There are two possible ways to improve the measurement accuracy:

- First, instead of a *single* measurement, we can make *repeated* measurements; the additional information coming from these additional measurements can improve the accuracy of the result of this series of measurements.
- Second, we can replace the *current* measuring instrument with a *more accurate* one; correspondingly, we can use a more accurate (and more expensive) measurement procedure provided by a measuring lab – e.g., a procedure that includes the use of a higher quality reagent.

In general, we can combine these two ways, and make *repeated* measurements with a *more accurate* measuring instrument. What is the appropriate trade-off between sample size and accuracy? This is the general problem that we address in this paper.

1. General formulation of the problem

We often need more accurate measurement procedures. Measurements are never 100% accurate, there is always a measurement inaccuracy.

Manufacturers of a measuring instrument usually provide the information about the accuracy of the corresponding measurements. In some practical situations, however, we want to know the value of the measured quantity with the accuracy which is higher than the guaranteed accuracy of a single measurement.

Comment. Measurements are provided either by a *measuring instrument* or, in situations like measuring level of pollutants in a given water sample, by a *measuring lab*. Most problems related to measurement accuracy are the same, whether we have an automatic device (measuring instrument) or operator-supervised procedure (measuring lab). In view of this similarity, in the following text, we will consider the term "measuring instrument" in the general sense, so that the measuring lab is viewed as a particular case of such (general) measuring instrument.

Two ways to improve the measurement accuracy: increasing sample size and improving accuracy. There are two possible ways to improve the measurement accuracy:

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In general, we can combine these two ways, and make *repeated* measurements with a *more accurate* measuring instrument.

Problem: finding the best trade-off between sample size and accuracy. What guidance shall we give to an engineer in this situation? Shall she make repeated measurements with the original instrument? shall she instead purchase a more accurate measuring instrument and make repeated measurements with this new instrument? How more accurate? how many measurement should we perform? In other words, what is the appropriate trade-off between sample size and accuracy?

This is the general problem that we address in this paper.

2. In different practical situations, this general problem can take different forms

There are two different situations which, crudely speaking, correspond to engineering and to science.

In most practical situations – in *engineering*, *ecology*, etc. – we know what accuracy we want to achieve. In *engineering*, this accuracy comes, e.g., from the tolerance with which we need to guarantee some parameters of the manufactured object. To make sure that these parameters fit into the tolerance intervals, we must measure them with the accuracy that is as good as the tolerance. For example, if we want to guarantee, e.g., the resistance of a certain wire does not deviate from its nominal value by more than 3%, then we must measure this resistance with an accuracy of at least 3% (or better).

In *ecological* measurements, we want to make sure that the measured quantity does not exceed the required limit. For example, if we want to guarantee that the concentration of a pollutant does not exceed 0.1 units, then we must be able to measure this concentration with an accuracy somewhat higher than 0.1. In such situations, our objective is to minimize the cost of achieving this accuracy.

In *science*, we often face a different objective:

- we have a certain amount of funding allocated for measuring the value of a certain quantity;
- within the given funding limits, we would like to determine the value of the measured quantity as accurately as possible.

In other words:

- In engineering situations, we have a fixed accuracy, and we want to minimize the measurement cost.
- In scientific situations, we have a fixed cost, and we want to maximally improve the measurement accuracy.

3. A realistic formulation of the trade-off problem

Traditional engineering approach. The traditional engineering approach to solving the above problem is based on the following assumptions – often made when processing uncertainty in engineering:

- that all the measurement uncertainties (“measurement errors”) are normally (Gaussian) distributed, with known standard deviations σ;
- that the measurement uncertainties corresponding to different measurements are independent random variables; and
- that the mean value Δ_s of the measurement uncertainty is 0.

Under these assumptions, if we repeat a measurement n times and compute the arithmetic average of n results, then this average approximates the actual value with a standard deviation $\frac{\sigma}{\sqrt{n}}$. So, under the above assumptions, by selecting appropriate large number of iterations n, we can make measurement uncertainties as small as we want.

This approach – and more general statistical approach – has been actively used in many applications to science in engineering problems; see, e.g., [5,6,15,19].

Limitations of the traditional approach. In practice, the distributions are often Gaussian and independent; however, the mean Δ_s (sometimes called “systematic error” in engineering practice) is not necessarily 0. Let us show this if we do not take this bias $\Delta_s \neq 0$ into account, we will underestimate the resulting measurement inaccuracy see, e.g., [14,16,18].

Indeed, suppose that we have a measuring instrument about which we know that its measurement uncertainty cannot exceed 0.1: $|\Delta x| \leqslant 0.1$. This means, e.g., that if, as a result of the measurement, we got the value $\tilde{x} = 1.0$, then the actual (unknown) value $x\ (= \tilde{x} - \Delta x)$ of the measured quantity can take any value from the interval $[1.0 - 0.1, 1.0 + 0.1] = [0.9, 1.1]$.

If the bias component of the measurement uncertainty is 0, then we can repeat the measurement many times and, as a result, get more and more accurate estimates of x. However, if – as is often the case – we do not have any information about