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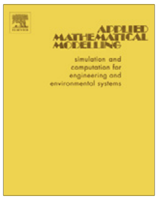

Optimum policy for a production system with major repair and preventive maintenance

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ABSTRACT

This study applies periodic preventive maintenance (PM) to a repairable production system with major repairs conducted after a failure. This study considers failed PM due to maintenance workers incorrectly performing PM and damages occurring after PM. Therefore, three PM types are considered: imperfect PM, perfect PM and failed PM. Imperfect PM has the same failure rate as that before PM, whereas perfect PM makes restores the system perfectly. Failed PM results in system deterioration and major repairs are required. The probability that PM is perfect or failed depends on the number of imperfect maintenance operations conducted since the previous renewal cycle. Mathematical formulas for expected total production cost per unit time are generated. Optimum PM time that minimizes cost is derived. Various special cases are considered, including the maintenance learning effect. A numerical example is given.

1. Introduction

Production policies have been investigated extensively with the aim of minimizing production costs [1–5]. To be globally competitive, manufacturers must have a cost-effective policy that considers operating costs and the effects of restorative activities, such as repairs and preventive maintenance (PM), on damaged production systems. Maintenance is conducted when equipment fails or as planned PM. This maintenance process requires planning, scheduling, control and deployment of maintenance resources. Devarun and Sandip [6] presented a multiple-criteria decision making methodology for selecting the optimal policy in the process industry. Sheu [7] evaluated maintenance policies in deteriorating production models. Lin et al. [8] examined maintenance and production and applied an imperfect production model to decrease the number of defects.

Notably, PM, which maintains production systems in top operating conditions, is performed regularly at pre-determined intervals to minimize operating costs and risk of catastrophic failure. A sequential PM policy requires that PM is performed at unequal intervals [9,10]. Bartholomew-Biggs et al. [11] examined the problem of scheduling sequential imperfect PM of some equipment. Whereas a PM is performed at fixed time intervals with a periodic PM policy [12]. The perfect PM model restores a system to "as good as new" after each PM action. Tseng [13] developed a perfect maintenance policy that increases the reliability of imperfect systems. Notably, imperfect PM models assume that, after PM, system failure rate is "as bad as before PM" [14–16]. The imperfect PM model developed by Nakagawa and Yasui [17] proposes that a PM model under which PM is either imperfect or perfect maintenance with probabilities of p and $1 - p$, respectively. However, the assumption of imperfect PM is not always true. In this study, we assume the probability of PM being perfect is related to the number of

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imperfect maintenance operations performed since the previous renewal cycle, and the probability of PM remaining imperfect does not increase.

Failed production system must be repaired. Cui et al. [18] considered random failure of production equipment inevitable. In response to each failure, a system is repaired and maintenance costs are minimized. Major repairs reset system failure incidence (since such repairs are perfect) and the system is restored to "as good as new". Numerous protective systems, such as circuit breakers, alarms, and protective relays, are maintained in this fashion, as described by Yang and Klutke [19].

In some cases, damage occurs following PM for human error in maintenance [20,21]. To model this phenomenon, this study considers three outcomes, similar to those proposed by Nakagawa and Yasui [17]. Outcome I involves imperfect PM only; outcome II involves perfect PM; and outcome III involves failed PM and a system requires major repairs. The remainder of this paper is described as follows. The cost of the production maintenance model with periodic PM is determined and an optimum policy is obtained in Section 2. Section 3 describes various special cases. Section 4 then provides a numerical result for these special cases. This study investigates the effects of these parameters on solutions. Finally, Section 5 presents concluding remarks.

2. General model

To construct the model, relevant notations are defined as follows:

$F(t)$	cumulative distribution function of a unit
$\overline{F}(t)$	survival function of a unit
$f(t)$	probability density function of a unit
T	time between PM for an operating unit
$*$	implies an optimum value
N	number of PM preceding the first perfect PM or failed PM occurs
$\overline{P}_j$	probability that the first j PM are imperfect maintenances; $\overline{P}_j = P(N > j)$
p_j	probability that PM is perfect or failed following the (j-1) imperfect PM; $p_j = P(N = j) = \overline{P}_{j-1} - \overline{P}_j = \overline{P}_{j-1}(1 - \overline{P}_j/\overline{P}_{j-1})$
$\{\overline{P}_j\}$	sequence of $\overline{P}_j, j = 0, 1, 2, \ldots$
q_j	probability that the jth PM is a imperfect PM; $q_j = \overline{P}_j/\overline{P}_{j-1}$
θ_j	probability that the jth PM is a perfect PM or a failed PM; $\theta_j = 1 - q_j$
$\theta_{1,j}$	the probability that a PM is classified as a perfect PM
$\theta_{2,j}$	the probability that a PM is classified as a failed PM; $\theta_j = \theta_{1,j} + \theta_{2,j}$
k	$\theta_{1,j}/\theta_{2,j}$; $k > 1$
R_{mf}	cost of each repair for maintenance failure
R_m	cost of each PM
R_f	cost of each repair for actual failure
R_p	operating cost per unit time
$H(t)$	$\left[\sum_{j=1}^{\infty}(\overline{P}_{j-1} - \overline{P}_j) jf(jt)\right]\Big/\left[\sum_{j=1}^{\infty}(\overline{P}_{j-1} - \overline{P}_j) j\overline{F}(jt)\right]$; [weighted sum of pdf $f(jt)$]/[weighted sum of Sf $\overline{F}(jt)$].
μ	the mean time between failure of the unit; $\int_0^{\infty}\overline{F}(t)dt$
$v(t)$	$\left(\sum_{j=1}^{\infty}(\overline{P}_{j-1} - \overline{P}_j) jf(jt)\right)\Big/\left(\sum_{j=1}^{\infty}\overline{P}_{j-1} jf(jt)\right), 0 < v(t) \leqslant 1$.
r	the learning rate
L	$R_f\Big/\left[\mu\left(R_f - \frac{R_{mf}}{1+k} - \frac{R_m}{v(\infty)}\right)\right]$
$C(T; \{\overline{P}_j\})$	The expected total production cost per unit time
Y	time between two successive renewal processes

In practice, periodic PM is performed to decrease the probability of future system failure and PM intervals are scheduled such that total production cost is minimized. This study considers a generalized PM model that has three outcomes after PM: imperfect PM, perfect PM, and failed PM. After PM, maintenance workers can perform periodic testing for abnormalities. In some cases, test results confirm that vigilance cannot be relaxed; otherwise, problems will recur. To incorporate these cases, the probabilities of perfect PM and failed PM in the proposed system depends on the number of PM operations conducted since the last renewal cycle.

The model is based on the following assumptions.

1. The original system begins operating at time 0.
2. A system has three PM types at time $j \cdot T (j = 1, 2, \ldots)$, based on outcomes.
    - Imperfect PM: a system has the same failure rate as before PM, the probability of which is $q_j = \overline{P}_j/\overline{P}_{j-1}$.
    - Perfect PM: a system is as good as new, the probability of which is $\theta_{1,j} = \frac{k}{1+k}(1 - q_j)$.