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Optimum preventive maintenance policies for systems subject to random working times, replacement, and minimal repair ☆

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ABSTRACT

This paper proposes, from the economical viewpoint of preventive maintenance in reliability theory, several preventive maintenance policies for an operating system that works for jobs at random times and is imperfectly maintained upon failure. As a failure occurs, the system suffers one of two types of failure based on a specific random mechanism: type-I (repairable) failure is rectified by a minimal repair, and type-II (non-repairable) failure is removed by a corrective replacement. First, a modified random and age replacement policy is considered in which the system is replaced at a planned time T, at a random working time, or at the first type-II failure, whichever occurs first. Next, as one extended model, the system may work continuously for N jobs with random working times. Finally, as another extended model, we might consider replacing an operating system at the first working time completion over a planned time T. For each policy, the optimal schedule of preventive replacement that minimizes the mean cost rate is presented analytically and discussed numerically. Because the framework and analysis are general, the proposed models extend several existing results.

1. Introduction

Almost all systems deteriorate owing to age and usage, and experience stochastic failures during actual operation. Deterioration raises operating costs and produces less competitive goods. Moreover, consecutive failures are dangerous to the whole system, so timely preventive maintenance is beneficial for supporting normal and continuous system operation. For these reasons, the development of various maintenance policies that seek the optimal decision models for reducing operating costs and the risk of a catastrophic breakdown is an important research topic for reliability engineers. In the past four decades, preventive maintenance models have generated increasing interest in reliability research. Some recent and related applications were introduced in Chang, Sheu, Chen, and Zhang (2011), Chang, Sheu, and Chen (2013a), Xia, Xi, and Zhou (2012), and Xu, Chen, and Yang (2012).

Age replacement policy (ARP) is a well-known preventive replacement model: an operating system is replaced at age T or at failure, whichever occurs first (Barlow & Hunter, 1960). In reality, it is not always possible to replace a failed system, and Barlow and Hunter (1960) introduce further the notice of periodic replacement with minimal repair for any intervening failures. In the literature relating to maintenance strategy upon failure, a system is typically assumed to be restored to a condition either "as good as new" (or simply replacement), or "as bad as old" prior to failure (i.e., minimal repair). This assumption seems not to be realistic, as discussed in Pham and Wang (1996). A choice between replacement and minimal repair is often based on some random mechanism. Brown and Proschan (1983) considered an imperfect repair model in which, upon failure, the system is replaced with probability p and is minimally repaired with probability $q(=1-p)$. Pham and Wang (1996) called such a repair mechanism an imperfect maintenance with (p, q) rule. This imperfect maintenance model has been extended and applied in reliability research, and some recent applications can be found in Chang, Sheu, and Chen (2010, 2013b) and Chen (2012). Other treatment models for imperfect maintenance have been proposed in the past from different perspectives, the most relevant efforts among them being the probabilistic approach (Block, Broges, & Savits, 1985; Brown & Proschan, 1983; Chang, Sheu, Chen, and Zhang, 2011; Chang, Sheu, and Chen, 2013a), the improvement factor method (Nakagawa, 1988), the virtual age model (Kijima, 1989; Kijima, Morimura, & Sujuki, 1988), the cumulative damage shock model (Kijima & Nakagawa, 1991; Zhao, Nakagawa, & Qian, 2012), and the other applied models (Huang, Lin, & Ho, 2013; Liu, Huang, & Wang, 2013). In this paper, we are concerned with modifying ARP by using the imperfect maintenance with (p, q) rule.

Unless otherwise specified, T is taken to be a constant time in ARP, and the optimum ARP is nonrandom for an infinite span

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(Brown & Proschan, 1965). However, some systems in offices and industries often execute jobs or computer procedures successively. For such systems, it would not be suitable to maintain or replace them in a strictly periodic fashion, because operational suspension for jobs may cause production losses to different degrees (Nakagawa, 2005, p. 245). When a job has a variable working cycle or processing time, it would be better to do maintenance or replacement after it has completed its work and process (Sugiura, Mizutani, & Nakagawa, 2004). If a system is replaced only at random times as its working times, the policy can be called a random replacement policy (Brown & Proschan, 1965). Early investigation into random replacement policies can be found in Yun and Choi (2000) and Stadje (2003). However, it has been assumed in many policies that the system is maintained or replaced preventively at a unique time scales, such as age, operating period, usage number, and damage level. In reliability applications, reliability and maintenance of systems are often measured using combined scales. Maintenance models with two time scales, such as age and usage number, age and failure number, and the like, have been discussed (Nakagawa, 2008, p. 149). In addition, combining an age replacement with random times in place of the traditional maintenance with constant time T was proposed for widespread practical application. Recently, Chen, Mizutani, and Nakagawa (2010) have looked into an operating system that works at successive random times and its age replacement policies. Chen, Nakamura, and Nakagawa (2010) consider replacement and maintenance policies for an operating system that works at random times and undergoes minimal repair at failures. However, an imperfect maintenance activity based on some random mechanism may be used to improve their works in reliability applications.

Two failure mechanisms (repairable and non-repairable) based on a model developed by Brown and Proschan (1983) are considered in this paper. First, a modified random and age replacement policy is considered in which the system is preventively replaced at a planned time T or at a random working time and is correctively replaced at any non-repairable failure, whichever occurs first. All repairable failures are rectified by a minimal repair. Next, as one extended model, the system may work continuously for N jobs with random working times. Finally, as a practice situation, it might be unrealistic to replace an operating system during the middle of some working time, even when the scheduled replacement time has arrived, but it might be wise to replace it at the first completion of working time over a planned time T. The optimum preventive replacement schedule for each modified model can be determined explicitly by minimizing its mean cost rate.

The main objective of this paper is to address the optimization of the modified random and age replacement policy for a system with imperfect maintenance quality. This research extends the works considered by Chen, Mizutani, and Nakagawa (2010) and Chen, Nakamura, and Nakagawa (2010) in which a system performs only either replacement or minimal repair at failure. However, a choice between replacement and minimal repair at failure is often considered in practical situations. We further combine their models via the imperfect maintenance with (p, q) rule that has been used generally in maintenance and replacement strategies. It can be seen that our models are the generalized research of random and age replacement policy in reliability application, which can contain two extreme cases of Chen, Mizutani, and Nakagawa (2010) and Chen, Nakamura, and Nakagawa (2010). In this paper, the mean cost rate models are derived analytically, the bivariate optimal solutions which minimize them are determined theoretically and computed numerically. Furthermore, a minimal repair/replacement decision based on the repair costs at failure is presented in the numerical example. The remainder of this paper is organized as follows. Section 2 presents the imperfect maintenance models of these preventive maintenance policies for an operating system, develops the mean cost rate functions, and focuses on the optimization of replacement schedules. In Section 3, a computational example is given to illustrate the applications of these preventive maintenance models. Finally, Section 4 presents our conclusions.

2. Preventive maintenance models

2.1. Model A

This model assumes a modified preventive maintenance policy for a system in which repair, maintenance, and replacement take place according to the following scheme.

(1) A new system with a failure time X begins to operate at time 0. When X has a general distribution $F(t)$ and probability density function $f(t)$, then the failure rate $r(t) \equiv f(t)/\overline{F}(t)$ is assumed to increase to $r(\infty) \equiv \lim_{t\to\infty} r(t)$ where $\overline{\Phi}(t) \equiv 1-\Phi(t)$ for any function $\Phi(t)$. A preventive replacement is planned to be conducted when the system reaches age T.
(2) It is assumed that system failure at time t can be of two types: a type-I failure (repairable or minor) occurs with probability q and is corrected by a minimal repair, whereas a type-II failure (non-repairable or catastrophic) occurs with probability $p(=1-q)$ and requires a corrective replacement. Note that the system failure rate $r(t)$ is not disturbed by any minimal repair.
(3) It is assumed that Y is the random time of the system with a general distribution $G(t)$ and does not take into account any actual failures (i.e., Y is independent of X) (Brown & Proschan, 1965, p. 72). It would be necessary to replace a system at random times as its working times in cases where the working time becomes large (Nakagawa, 2005, p. 245). Another preventive replacement is performed at the completion of the working time.
(4) In summary, the system is replaced at time T, Y, or immediately after any type-II failure, whichever occurs first, where T is a constant and Y is a random variable with distribution $G(t)$. The replacement process of model A is shown in Fig. 1.
(5) Repairs and replacements are completed instantaneously. After a replacement, the system becomes brand new and resets to time 0. A renewal cycle is defined as the time interval between two consecutive replacements.

Suppose that Z is the waiting time until the first type-II failure and hence is also independent of Y. From Beichelt (1993), the survival function of Z is directly obtained

$$\overline{F}_p(t) \equiv P(Z > t) = \exp(-p\Lambda(t)), \tag{1}$$

where the cumulative hazard $\Lambda(t) \equiv \int_0^t r(u)\mathrm{d}u = -\ln \overline{F}(t)$ is the mean number of failures that occur in [0, t]. The corresponding probability of each replacement situation in a renewal cycle can be derived as below (refer to Nakagawa, 2005, p. 246). The probability that the system is replaced at age T is

$$P(Z > T, Y > T) = \overline{F}_p(T)\overline{G}(T), \tag{2}$$

the probability that it is replaced at random time Y is

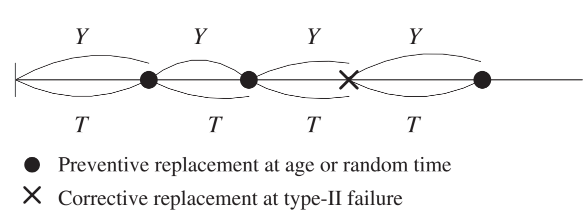

Fig. 1. Process of preventive and corrective replacement in Model A.