



## A computational approach to pricing a bank credit line

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### ABSTRACT

Using trended Brownian motion to characterize borrower cash needs over time, we are able to derive a probability density function for the time to depletion of a bank credit line as well as the likelihoods for the time to exhausting the sources of liquidity that fund the loan. Armed with these analytic results, we solve for the credit line mark-up rate and the configuration of stored liquidity that maximizes the bank's intertemporal expected profits from the loan. The optimality conditions produce a system of integral differential equations whose solutions we then simulate over a host of scenarios.

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### 1. Introduction

A bank loan commitment is a lending contract that provides a line of credit to a borrower.<sup>1</sup> The size of the commitment is the maximum amount that can be borrowed by the loan recipient over the life of the contract. Customer usage of the line of credit is known as the “takedown”; borrowing against the line of credit can also be called “drawing down” the loan account. The contract has a maturity, known as the “duration” of the loan commitment. Commercial banks typically engage in “LIBOR plus” pricing where what is added to LIBOR is the bank's mark-up rate which is fixed over the life of the loan contract. In addition, banks often impose a variety of fees on a loan commitment.<sup>2</sup> These fees include a commitment fee, an annual service fee and a non-usage fee. The commitment fee is an up-front fee imposed at the time the loan commitment is made. The bank may charge an annual service fee on the amount borrowed every year. A non-usage fee is often charged on the amount of credit that is available but not used by the borrower. A bank can charge the cus-

tomers any combination of these fees; some banks may charge a borrower all three fees, while others may not charge any of these fees.<sup>3</sup>

The existence of loan commitments can be explained by the benefits it provides to both the borrower and the lender. Commitments allow the borrower to transfer interest rate risk to the creditor. Campbell (1978) provides the first model to document this transfer as he solves for the fee structure of a fixed rate loan commitment that maximizes the bank's expected utility. Boot et al. (1987) demonstrate that loan commitments provide incentives to the borrower to choose safer projects and put forth greater effort. Both choices result in higher profits for the borrower on average. Thakor (1989) argues that loan commitments allow the borrowing firm to overcome the pre-contract asymmetric information problems that plague debt financing.<sup>4</sup> According to Maksimovic (1990), borrowers can gain an advantage over competitors who acquire funds in the spot market by purchasing the option to own a loan commitment. This option allows firms that operate under imperfect competition to determine the method of financing before its production decision is made. Shockley (1995) shows that firms

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<sup>1</sup> The academic and practitioner literatures use the terms “loan commitment”, “credit line” and “revolver loan” interchangeably. In Ergungor's (2001) review of the literature, the author places literature on credit lines alongside other loan commitment papers, demonstrating that there is no distinction between the two. Campbell (1978) uses “revolving credit agreement” and “line of credit” synonymously.

<sup>2</sup> Early theoretical literature like Campbell (1978) uses fixed rates for the mark-up fee; recent empirical work focuses on loan commitments with a variable rate component.

<sup>3</sup> Greenbaum and Venezia (1985) detail the spectrum of loan commitment contract fees and the extent to which they are used.

<sup>4</sup> Thakor maintains that the likelihood of firm success is known by the borrowers, not the lenders. Companies that are willing to acquire funds via a loan commitment disclose information about their “type” (or caliber). If a bank understands the borrower's type, it can propose a combination of fees that will maximize expected utility of both parties. The author shows that the loan commitment allows for sorting between firms; the best firms (better investment opportunities) will choose the loan commitment and the weaker firms (with the poor investment opportunities) will borrow in the spot market.

that have more loan commitments than spot loans will benefit from a lower cost of debt, which leads to greater capacity to borrow.<sup>5</sup>

Loan commitments also provide benefits to the issuing bank. Credit line contracts assist banks in forecasting future loan demand according to Greenbaum et al. (1991). The authors show that the fee structure of the loan commitment gives the customer incentives to disclose information about its anticipated loan takedown. In exchange, the bank offers lower borrowing rates but reaps extra profits gained from the customer's cooperation.<sup>6</sup> In their analysis of financial contracting, Boot et al. (1993) illustrate how loan commitments allow banks to enhance its reputational capital. Banks that honor a financial contract, even if a borrower encounters financial distress, do so at a short-term financial cost, but they enjoy a long run gain in reputation for honoring its lending agreements. Thakor and Udell (1987) demonstrate how banks that offer loan commitments can acquire private information about borrowers that lenders in the spot market cannot obtain. In their discussion of loan commitments as the optimal form of bank financing, the authors show that information about the customer behavior (or type) is revealed by the choice of contract.<sup>7</sup> Avery and Berger (1991) find empirical evidence suggesting banks with large amounts of credit line loans (relative to traditional lending) have fewer defaults compared to their peers with fewer credit line commitments.<sup>8</sup>

In order to price a line of credit the bank must consider the implications of borrower needs for the timing and the magnitude of the loan's takedown. Intertemporal bank revenues and funding costs clearly depend upon the nature of the stochastic drawdown of the loan commitment. If the credit line is entirely depleted within a few days of its inception then usage fees will generate sizable revenues while non-usage fees will be insignificant. The assets used to fund the loan takedown will give rise to very little interest income and asset conversion costs will be fully realized. If a dramatic loan takedown occurs near the maturity of the credit line then the bank's usage fees will be small while the non-usage revenue will be relatively large. Assets, held to fund borrower takedown, will generate sizable interim income but suffer significant liquidation costs at the end of the loan contract. If the credit line goes nearly unused through the life of the loan commitment then usage fees will be negligible while the bank will enjoy increased income from non-usage fees. Interest earned from loan funding sources will be enhanced and asset liquidation costs will be small. Clearly if expected profits are to be maximized across time then

the likelihood of these scenarios, and others, must be taken into consideration by the bank.<sup>9</sup>

We model the cash needs of the borrower as trended Brownian motion,

$$p(c_0, c; t) = \frac{1}{\sigma\sqrt{2\pi t}} e^{-(c-c_0-\mu t)^2/2\sigma^2 t},$$

so that the loan applicant's cash demands at any point in time "t", are normally distributed. If borrower demands are expected to grow rapidly then the parameter  $\mu$  will be relatively large. If there is a great deal of uncertainty regarding the loan recipient's financial needs then  $\sigma$  will be large. Using  $p(c_0, c; t)$ , we are able to characterize the expected borrowing from the bank over time. In addition, the statistical properties of trended Brownian motion allow us to derive a probability density function for the time to depletion of the loan account, as well as likelihoods for the time to exhausting the respective bank assets that fund the loan takedown.

## 2. Time to loan depletion

For our model, we will assume that the cash needs of the borrower  $C(t)$  follow a Brownian motion process with a drift of  $\mu$  where  $\mu > 0$ . The borrower's cash needs represent a dollar for dollar takedown of the credit line and the time to loan depletion  $T_W$  of  $C(t)$  is characterized by  $C(T_W) = W$  where  $W$  is the size of the loan commitment. If  $C(t) < W$  then  $t < T_W$ . That is, we let  $p(c_0, c; t)$  be the probability that  $C(t) = c$  and that the process does not reach the point of loan depletion in the time interval  $(0, t)$ .<sup>10</sup> Consequently,

$$P\{C(t) < W \text{ for } t < T_W | C(0) = c_0\} = \int_{-\infty}^W p(c_0, c; t) dc \\ \equiv P(c_0, W; t), \quad (1a)$$

is the probability that the time to depletion of  $W$  has not occurred by  $t$ , we have

$$P(c_0, W; t) = \text{prob}(T_W \geq t). \quad (1b)$$

So that

$$P(c_0, W; t) = \int_{-\infty}^W p(c_0, c; t) dc = 1 - G(t; c_0, W), \quad (1c)$$

where  $G(t; c_0, W)$  is the cumulative probability of the time to the loan's depletion.

Consequently,

$$\frac{\partial P(c_0, W; t)}{\partial t} = -G'(t; c_0, W), \quad (2a)$$

or

$$-\frac{\partial P(c_0, W; t)}{\partial t} = g(t; c_0, W), \quad (2b)$$

where  $g(t; c_0, W)$  is the marginal probability density function of  $T_W$ .

<sup>5</sup> Shockley presents empirical evidence from a sample of 300 industrial firms that greater loan commitment usage (relative to spot loans) results in greater debt-to-asset ratios and a lower cost of borrowing on average. The uniqueness of the loan commitment contract, versus other debt contracts, reduces the lender's uncertainty about the borrower's behavior and allows the bank to offer cheaper funds to commitment customers.

<sup>6</sup> Many papers in the loan commitment literature discuss the structure, incidence, and rationale of loan fees. James (1981) studies the borrower's choice between paying commitment fees and maintaining compensating balances with the lender as part of the loan contract. In solving for the firm's optimal level of compensating balance deposits, James finds that fees and balances are not perfect substitutes. Ho and Saunders (1983) investigate how banks manage interest rate risk as a result of entering into fixed rate loan commitment contracts. Since a bank cannot perfectly hedge this risk with futures contracts, it must charge a reservation fee (also known as a commitment fee) as compensation for risk-sharing. In their explanation of why the loan commitment is the optimal debt contract, Berkovitch and Greenbaum (1991) explain and justify the existence of different contract fees. Loan commitment fees help alleviate information problems between bank and borrower. In addition, the authors explain how loan commitments reduce investment distortions, such as the "underinvestment problem".

<sup>7</sup> Customers that plan on using the full amount of available funds in the commitment will opt for a high non-usage fee, and customers who will use the commitment sparingly will choose a low non-usage fee.

<sup>8</sup> Avery and Berger build a related model confirming that banks issuing credit lines are subject to the moral hazard of borrowers and that the lender's ability to ration customers based on risk is crucial to bank profitability.

<sup>9</sup> This literature also includes research on loan commitment valuation, or the pricing of the actual debt contract itself. The ability to value a credit line, which is an off-balance sheet asset, is an important tool for a research analyst who wants to accurately price a bank's equity or understand the bank's risk exposure. Thakor et al. (1981) make the first attempt to value loan commitments. In this paper, the authors develop a model for valuing variable rate bank loan commitments within the framework of the Black and Scholes pricing methodology. Using numerical simulations, the authors examine the sensitivity of these contracts to changes in market interest rates.

<sup>10</sup> The borrower's cash needs have unbounded support in this paper. Consequently, problems of negative cash needs can arise. Throughout the paper, the non-negative constraint is ignored for simplicity. The probability of a borrower having negative cash needs can be made arbitrarily small by the appropriate choice of the underlying statistical parameters that we use to characterize  $p(c_0, c; t)$ .

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