Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Computers and Mathematics with Applications 51 (2006) 189–198
An International Journal
computers & mathematics with applications
www.elsevier.com/locate/camwa

Properties of a Job Search Problem on a Partially Observable Markov Chain in a Dynamic Economy

TŌRU NAKAI
Department of Economic Engineering, Faculty of Economics
Kyushu University, Hakozaki 6-19-1, Fukuoka 812-8581, Japan
nakai@en.kyushu-u.ac.jp

Abstract—This paper observes a job search problem on a partially observable Markov chain, which can be considered as an extension of a job search in a dynamic economy in [1]. This problem is formulated as the state changes according to a partially observable Markov chain, i.e., the current state cannot be observed but there exists some information regarding what a present state is. All information about the unobservable state are summarized by the probability distributions on the state space, and we employ the Bayes' theorem as a learning procedure. The total positivity of order two, or simply TP_2, is a fundamental property to investigate sequential decision problems, and it also plays an important role in the Bayesian learning procedure for a partially observable Markov process. By using this property, we consider some relationships among prior and posterior information, and the optimal policy. We will also observe the probabilities to make a transition into each state after some additional transitions by employing the optimal policy. In the stock market, suppose that the states correspond to the business situation of one company and if there is a state designating the default, then the problem is what time the stocks are sold off before bankrupt, and the probability to become bankrupt will be also observed.

Keywords—Partially observable Markov chain, Total positivity, Job search problem.

1. INTRODUCTION

This paper will observe a job search problem on a partially observable Markov chain and will consider the probability to make a transition into each state after some additional transitions by employing the optimal policy. This is one of the optimal stopping problems and can be considered as an extension of a job search in a dynamic economy discussed by Lippmann and MacCall [1]. For instance, in economics, we consider that the conditions of economy are divided into some classes, and assume them to be getting worse. Let us assume that these conditions are not directly observable. That is, it cannot be known which one of these classes it is now, but there is some information regarding what a present class is. When each state of this Markov chain corresponds the class of the economy, we suppose that the wages of a job are a random variable depending on these classes. Differing from the case in [1], the state changes according to a partially observable Markov chain. On the other hand, in the stock market, we consider these classes to correspond to the business situation of one company, i.e., these conditions can be

This research was partially supported by the Grant-in-Aid for Scientific Research of the Japanese Ministry of Education, Science, Sports and Technology. The author wishes to thank the referees for their helpful comments.

0898-1221/06/$ - see front matter Typeset by $\mathcal{AMS}$-TeX
doi:10.1016/j.camwa.2005.11.019

estimated through the stock price of this company in the stock market. Since the stock price is observable, the problem is what time the stocks are sold off. For a job search problem in which the current state is observable, it is known that the maximization is achieved by classifying all possible job offers into two mutually exclusive classes, and the wage of a job offer that separates these two classes is called the reservation wage. It is not, however, always true for this problem since the state of the chain is unobservable for the decision maker.

All information about the unobservable state is summarized by probability distributions on the state space, and we employ the Bayes' theorem as a learning procedure. The total positivity of order two, or simply TP_2, is a fundamental property to investigate sequential decision problems, and it also plays an important role in the Bayesian learning procedure for a partially observable Markov process. By using this property, we consider some relationships among prior and posterior informations, the optimal policy and the probabilities to make a transition into each state. The properties of this TP_2 are also investigated by Karlin and McGregor [2], Karlin [3], Karlin and Rinott [4], and by others regarding the stochastic processes.

In order to observe these probabilities when we employ the Bayes' theorem as a learning procedure, we will start to reconsider a job search in a dynamic economy to compare the properties of this problem on a partially observable Markov chain. In Section 2, we summarize the properties of a job search problem when the state of the chain is directly observable. It will be shown that the probabilities to make a transition into each state after some additional transitions is TP_2. In Section 3, we will investigate the probabilities to make a transition into each state when the state changes according to a partially observable Markov chain. We will also observe the similar probabilities by employing the optimal policy of the job search problem. Suppose that the State i represents the class of the business situation of one company ($i \in \{1, 2, \dots, K\}$), and we specially suppose that the State K designates the default. Then the problem is what time the stocks are sold off before bankrupt, and the probability to become bankrupt is also observed by these considerations.

2. JOB SEARCH IN A DYNAMIC ECONOMY

2.1. Optimal Policy and the Expected Reward

Consider a finite state Markov chain with the state space $\{1, 2, \dots, K\}$ with the transition probability $\mathbf{P} = (p_{ij})_{i,j=1,2,\dots,K}$. The job search is a problem to find a job in order to maximize the expected reward without recall, and the jobs appear one at a time in sequential order. When each State i of this Markov chain corresponds the class of economy, let X_i be a nonnegative real valued random variable representing the wage of a job associated to this condition ($i = 1, 2, \dots, K$). When the decision-maker knows what a present state is, Lippmann and MacCall [1] considered this problem in a dynamic economy under two conditions (1) and (2):

(1) X_i is stochastically increasing in i, i.e., $F_1(x) \geq F_2(x) \geq \cdots \geq F_K(x)$ for all x,
(2) $\sum_{j=k}^{K} p_{ij}$ is increasing in i for all k ($k = 1, 2, \dots, K$).

In this paper, we treat this job search problem under an uncertainty condition, i.e., the state of this chain cannot be observed directly. Since the TP_2 plays an important role in the Bayesian learning procedure, we introduce two assumptions concerning the transition probability and the distribution function of X_i as Assumptions 1 and 2 ($i = 1, 2, \dots, K$), which are the differences to the Lippmann and MacCall's case. These nonnegative random variables X_i are absolutely continuous with the density function $f_i(x)$ ($i = 1, 2, \dots, K$). It is possible to generalize this problem for a partially observable Markov process as is observed by Nakai [5], and also apply it to the sequential decision problems (see [6–8] and so on). In Definition 1, we introduce a stochastic relation among random variables defined on a complete separable metric space with total order $\geq$.