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Journal of Monetary Economics 54 (2007) 2269–2290

Journal of MONETARY ECONOMICS

www.elsevier.com/locate/jme

A generalized volatility bound for dynamic economies[☆]

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Received 22 November 2006; received in revised form 14 June 2007; accepted 21 June 2007

Abstract

We develop a generalization of the Hansen–Jagannathan (1991) volatility bound that (i) incorporates the serial correlation properties of return data and (ii) allows us to calculate a *spectral* version of the bound. This generalization enables us to judge whether models match important aspects of the data in the long run, at business cycle frequencies, seasonal frequencies, etc. Our bound permits evaluation of models without requiring their explicit solution in a way that respects the dynamic implications of the fundamental component of the models, namely, the Euler equation that links asset returns to the intertemporal marginal rate of substitution.

JEL classification: E2; G2

Keywords: Volatility bound; Spectral; Asset-pricing

[☆] We gratefully acknowledge the support provided for this research by the National Science Foundation under Grants SES-0082237 and SES-0082230. Otrok also thanks the Bankard Fund for Political Economy for financial support. We are grateful for comments by Erzo Luttmer and Tim Cogley; they helped shape our thinking on this project. We also thank seminar participants at the Federal Reserve Banks of Atlanta, Minneapolis and Richmond, Yale University, the University of Pittsburgh, the University of Quebec-Montreal, Iowa State University, and the Society for Economic Dynamics meetings in 2001.

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doi:10.1016/j.jmoneco.2007.06.028

1. Introduction

Empirical evaluation of dynamic structural models has a long history in economics. In dynamic general equilibrium frameworks with linear–quadratic preferences, Hansen and Sargent (1980) computed linear decision rules explicitly and linked theory with measurement using the likelihood function. As they showed, a frequency domain approximation to the likelihood may be used in model assessment, in which case the fit of a model is judged by how well its spectral density matches the corresponding spectral density computed from the data. The procedure, unfortunately, rejects practically all models. In circumstances where closed-form expressions for decision rules are not available, Hansen (1982) and Hansen and Singleton (1983) used the Generalized Method of Moments (GMM) to formally estimate and evaluate dynamic models using a subset of the model implications for the data. Tests based on the procedure are less demanding than those based on the full likelihood, though few models pass them. Hansen and Jagannathan (1991) proposed a still less restrictive test that generalizes the variance bounds developed by LeRoy and Porter (1981) and Shiller (1981). They showed how to use asset return data to derive a lower bound on the volatility of a representative consumer's intertemporal marginal rate of substitution (IMRS). A model is said to be consistent with the data if the volatility of the IMRS implied by the model is greater than the Hansen–Jagannathan (HJ) volatility bound.

While the HJ test dismisses many models for violating the volatility bound, many others do satisfy the bound. Cochrane and Hansen (1992), for instance, argued that for reasonable parameterizations of time non-separable preferences and of state non-separable preferences, the consumption-based asset-pricing models do satisfy the HJ bound. Tallarini (2000) modified the preferences in the standard business cycle model to allow for non-separabilities across states and showed that the model is consistent with asset return data using the HJ bound.

The HJ bound depends on three types of asset return moments: means, variances and contemporaneous correlations. In this paper, we develop a generalized volatility bound that (i) incorporates the serial correlation properties of return data and (ii) allows us to calculate a *spectral* decomposition of the bound. This enables us to judge whether models fail to match important aspects of the data in the long run, at business cycle frequencies, seasonal frequencies, high frequencies, etc. Our evaluation of models is also based solely on the Euler equation that links the asset returns to the IMRS. This Euler equation governs intertemporal decisions, and hence the propagation of economic fluctuations—so a spectral (i.e., temporal) bound is especially useful in evaluating the model. Specifically, we can identify the frequencies at which a model violates the necessary conditions.

Technically, to derive the bound, Hansen and Jagannathan projected the model IMRS onto the space of contemporaneous asset returns and utilized only a *necessary* condition associated with dynamic models, namely the intertemporal Euler equation. Our generalization involves projecting the model IMRS onto the space of current, past, and future returns. As in Hansen and Jagannathan (1991), the projection involves a covariance between the IMRS and returns, a covariance which is given by Euler equation of the model and which is the sole implication used in the derivation of our bound. We show that the *variance* of the model IMRS must exceed the *variance* of the projection. Because of the