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A note on public debt, tax-exempt bonds, and Ponzi games

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ABSTRACT

By issuing tax-exempt bonds, the government can incur debt and never pay back any principal or interest, even if the economy without public debt evolves on a dynamically efficient growth path. The welfare effects of such a Ponzi type borrowing scheme are mixed. The current young will unambiguously benefit. Depending on preferences and the aggregate technology, a finite number of subsequent generations may also benefit. However, the welfare of all generations thereafter will be lower than in the economy without public debt.

1. Introduction

The Diamond (1965) overlapping generations model may generate competitive equilibria in which the growth rate of the labor force exceeds the long-run return on capital. In these cases the economy is said to be dynamically inefficient, and the government can play a so-called Ponzi game. That is, the government can issue bonds and roll over interest and principal from period to period by perpetually issuing new bonds to render debt service. Such a Ponzi game is beneficial as it removes overaccumulation of capital associated with dynamic inefficiency.[1]

As has been demonstrated by Uhlig (1998), the government can even run a Ponzi game if the economy without public debt is dynamically efficient. This becomes possible when the return on private bonds or equity is taxed and the government issues tax-exempt bonds.[2] Uhlig's analysis confines attention to a comparison of welfare in various steady state equilibria. The present note, in contrast, fully characterizes the dynamics of the model economy. It demonstrates that unlike a traditional Ponzi game, the welfare effects of a Ponzi game based on the issuance of tax-exempt bonds are mixed. The current young will unambiguously benefit. In addition, a finite number of subsequent generations may also benefit, depending on preferences and the aggregate technology. Thereafter, however, welfare is lower than in the economy without public debt.

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[1] Many authors have studied the properties of Ponzi games (or bubbles) in dynamically inefficient economies. See, e.g., Tirole (1985), O'Connell and Zeldes (1988), Ball et al. (1998), Chalk (2000) and Wigger (2005).

[2] Some countries, e.g., China and India, exempt interest income on public bonds from income taxation. In the United States interest income on bonds issued by lower levels of government is exempted from federal income taxation. See Norregaard (1997) for a comprehensive survey on the ramifications of tax-exempting income on public bonds.

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2. The model

Consider an overlapping generations economy in which at each time t a new generation, referred to as generation t, is born and lives for two periods. In the first period of life individuals supply one unit of labor, consume, and save for old-age by purchasing one period bonds in the capital market. In the second period of life individuals retire and consume out of the proceedings of their savings. The size of each generation is an N multiple of its forerunner.

Utility of a generation t individual is assumed to be $u_t = \ln c_t + \beta \ln z_t$, where c_t and z_t are young- and old-age consumption and $\beta > 0$ is a discount factor. Young-age consumption is determined by $c_t = w_t - s_t$ and old-age consumption by $z_t = R^n_{t+1} s_t$, where w_t and s_t are the wage rate and the individual's saving at time t, and R^n_{t+1} is the net of tax rate of interest paid on one period (private or public) bonds at time $t+1$. Saving is determined by utility maximization and reads

$$s_t = \gamma w_t, \tag{1}$$

where $\gamma \equiv \beta/(1+\beta)$ is the marginal propensity to save out of labor income.

Competitive firms finance investment by issuing one period bonds and hire labor to produce a homogenous good which serves for both consumption and investment purposes. Aggregate production of all firms at time t is determined by $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ and $\alpha > 0$ are technological parameters, K_t is the capital stock at time t, and L_t is the labor force at time t which equals the size of generation t.

Each factor of production is paid its marginal product. Assuming that capital fully depreciates within one period, the gross rate of interest that competitive firms pay on one period bonds at time t thus reads

$$R_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} \tag{2}$$

and the wage rate at time t is given by

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha}. \tag{3}$$

The government imposes a capital tax at the rate τ, so that the net of tax interest rate that individuals receive on their savings at time t reads $R^n_t = (1-\tau)R_t$. By construction, the capital tax does not only apply to the interest received on private bonds, but also to the principal. This assumption will simplify the difference equation characterizing the dynamics of bond accumulation. As capital is assumed to fully depreciate within one period, the assumption implies that capital depreciation is not tax deductible.[3]

In addition to capital taxation, the government generates revenue by issuing bonds amounting to B_0 at time 0. Both, tax revenue and revenue by issuing bonds at time 0 are used for some (wasteful) expenditure that does not affect individual utility.

Rather than paying back the debt, the government rolls over interest and principal from period to period by issuing new bonds at each time t. If such a Ponzi game is feasible, the amount of bonds issued by the government at time $t+1$ will equal

$$B_{t+1} = R^b_{t+1} B_t, \tag{4}$$

where R^b_{t+1} is the rate of interest paid on one period public bonds.

In contrast to private bonds, public bonds are tax-exempt. Therefore, non-arbitrage in the bond market requires

$$R^b_t = (1-\tau) R_t. \tag{5}$$

Equilibrium in the capital market obtains, when aggregate savings equal the amount of public and private bonds. As the latter determine the stock of capital at time $t+1$, the capital market equilibrium condition may be written as

$$L_t s_t = B_t + K_{t+1}. \tag{6}$$

The evolution of the economy can be characterized by two difference equations determining the dynamics of public bonds and the capital stock. It is convenient to express both magnitudes in intensive form. Thus, let $k_t = K_t/L_t$ denote the capital stock per unit of labor and $b_t = B_t/Y_t$ the debt-GDP ratio at time t.[4] Then, straightforward manipulation of Eqs. (1) through (6) yields

$$k_{t+1} = \frac{1}{N}[\gamma(1-\alpha) - b_t] A k_t^{\alpha}, \tag{7}$$

$$b_{t+1} = \frac{(1-\tau)\alpha b_t}{\gamma(1-\alpha) - b_t}, \quad \text{for } t = 0, 1, \ldots \tag{8}$$

[3] Since private savings are completely inelastic with respect to interest income [see Eq. (1)], qualitatively similar results would be obtained, if the tax rate at time t applied to $R_t - 1$ rather than R_t.

[4] Expressing public debt in per GDP units (rather than expressing it in per labor units) facilitates the analysis as it allows to separate the bond dynamics from the dynamics of the capital stock per worker.