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Building life-cycle cost analysis due to mainshock and aftershock occurrences

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ABSTRACT

The objective of this paper is to develop formal stochastic expected financial loss estimation models over the lifetime of the building due to mainshocks and their subsequent aftershock sequences. Mainshocks are typically modeled as a homogeneous Poisson process with constant mean rate of occurrence, while the resulting aftershocks are modeled as a nonhomogeneous Poisson process with random magnitudes which has parameters (mainshock magnitude, m_m, and location) that are conditional on the random mainshock. The initial model to compute expected losses is the simplified homogeneous Poisson mainshock process and nonhomogeneous Poisson aftershock process with "immediate" repair of the building to the initial building state. We then develop a more general Markov and semi-Markov framework where we consider both Poisson and renewal processes for modeling mainshock occurrences with various building damage progression scenarios. Finally, we will incorporate the random aftershock losses into pre-mainshock financial loss estimation. The ability to compute the expected building life-cycle cost due to both mainshocks and aftershocks will be useful as an input to seismic decision making (both post- and pre-mainshock).

1. Introduction

Performance-based earthquake engineering (PBEE) has been an area of active research in the United States. The Pacific Earthquake Engineering Research Center (PEER) led an effort to develop a quantitative PBEE methodology which allows stakeholders to make better informed decisions by providing them with probabilistic descriptions of system-level performance metrics, or decision variables (*DV*s), such as fatalities, financial losses and downtimes. A consistent probabilistic framework is used to explicitly and rigorously quantify the inherent uncertainties and randomness in all variables (see [1,2]). Probabilistic information of *DV*s can be used by stakeholders to make better seismic-risk related decisions, typically based on an optimization procedure to minimize the expected levels of financial losses, fatalities or downtimes.

For example, [3] developed a formulation for rational (pre-mainshock) design criteria. Such a formulation provides a quantitative decision making procedure where we select optimal building designs based on the minimization of expected life-cycle cost, including the initial cost of design and construction and the cost of potential damage and failure during the building's life-span. Earthquakes are modeled as homogeneous Poisson processes in this study. Similarly, [4] presented a decision methodology based on expected life-cycle cost analysis as well, but they used a renewal model for earthquake occurrences instead. Only mainshocks are considered in both studies. Business disruption financial losses have not been explicitly taken into consideration as well.

However, the *DVs* may also be very dependent on the post-earthquake performance of (possibly mainshock-damaged) buildings in the aftershock environment where there is a significantly increased rate of earthquake occurrence (see [5,6]). The mean rate of aftershocks, which is mainshock-magnitude, m_m, dependent, decreases with increasing elapsed time t from the occurrence of the mainshock. Such elements deserve closer scrutiny as aftershocks could potentially pose excessive life-safety threat to building occupants and contribute significantly to financial losses and downtime, especially if the building has suffered structural damage after the mainshock with reduced capacity to resist potential future aftershocks. The ground motions from aftershocks also show the typically high event-to-event variability, implying the potential for larger motions from small magnitudes. The number, size, proximity and variability of aftershocks may represent a significant ground motion hazard.

Because of this potential for larger ground motions due to aftershocks, even buildings that have not been damaged by the mainshock have some likelihood of being damaged due to the occurrence of an aftershock. Mainshock-damaged buildings are even more susceptible to incremental damage due to aftershocks because their deteriorated structural capacities reduce the threshold of the ground motion intensity needed to cause further damage. Hence, further financial losses can result from the aftershock sequence that follows the mainshock.

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The financial losses will include one-time transition costs, or costs from further structural and nonstructural damage to the building due to the occurrence of an earthquake. The transition cost may also include the costs of evacuation of the occupants of a building. The financial losses due to an earthquake also include disruption or downtime costs due to the building's non-operability or limited functionality. The evacuation of building occupants from a building that has suffered significant structural damage during the mainshock may be necessary which can significantly increase financial losses due to loss of revenue. In some cases, unless the continued functionality of the building is essential to the building owner, it might take 2 years or more before re-opening of the mainshock-damaged building. [7] and [8] have documented examples of such cases of significant business disruption in mainshock-damaged buildings.

Because the performance of mainshock-damaged buildings in the aftershock environment may have such a significant impact on the post-quake functionality and economic consequences of an earthquake, aftershock considerations should perhaps also have substantial influence on pre-mainshock decision making. An example of pre-mainshock decision would be whether or not to retrofit prior to the occurrence of a mainshock or whether to expend additional funds to provide a structure with enhanced aftershock performance.

Thus, for seismic-risk management, financial losses in terms of transition and disruption losses due to both mainshocks and aftershocks need to be quantified. This information will serve as an input to post-quake decision making and pre-mainshock design decisions that are dependent on the post-quake functionality of structurally damaged buildings. The development of a methodology to compute the expected financial losses due to both aftershocks and mainshocks will be the focus of this paper.

We will derive the expected financial losses for both the homogeneous mainshock process as well as the nonhomogeneous aftershock process in the simplified Poisson process model. We will also relax the restrictions of the Poisson model by formulating a method of obtaining the expected financial losses for both the homogeneous mainshock process and the nonhomogeneous aftershock process embedded in the more general Markov process framework to describe the transitions from one building damage state to another. We also consider a (non-Poissonian) renewal mainshock process in the formulation of expected financial losses where we use an arbitrary inter-arrival time distribution for mainshock occurrences. Lastly, we will also incorporate the random aftershock losses into pre-mainshock financial loss estimation to be used as an input to pre-mainshock design analysis. A simple case study is also presented to illustrate the proposed formulation.

2. Poisson loss model

To begin, we consider a building in damage state i which could go to one of n damage states given the occurrence of an earthquake (either mainshock or aftershock). The intact state is typically denoted as damage state 1 and the collapse state is typically denoted as damage state n. A building in damage state i can go to damage state j with probability Q_{ij} given the occurrence of an earthquake. $Q_{ij} = 0$ for $i > j$ if we assume no repair operations, and $\sum_j Q_{ij} = 1$. The transition matrix, $\mathbf{Q}$, is shown in Eq. (1).

$$\mathbf{Q} = \begin{pmatrix} Q_{11} & Q_{12} & \cdots & Q_{1n} \\ \vdots & Q_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & Q_{(n-1)n} \\ 0 & \cdots & \cdots & 1 \end{pmatrix} \tag{1}$$

If an earthquake occurs and the building suffers additional damage and enters damage state j, where $j > i$, we assume that there is incurred a (random) transition cost of L_{ij} which is dependent on states i and j. While the building remains in damage state i, we assume that it incurs a constant (but random) disruption cost of R_i per unit time due to the downtime and limited functionality of the damaged structure. We further assume $E(L_{ij}) = l_{ij}$ and $E(R_i) = r_i$.

We first propose a simplified method to quantify the financial losses by using a Poisson model for earthquakes where the intensity function $\mu(t)$ can either be homogeneous for mainshocks (i.e., $\mu(t) = \mu$, a constant value independent of time) or nonhomogeneous for aftershocks (i.e., $\mu(t;m_m)$ is time-dependent and mainshock-magnitude dependent). For the formulation to follow, we suppress in the notation the dependence of the aftershock intensity function on m_m. $\mu(t)$ represents the instantaneous daily rate of earthquakes (both mainshocks and aftershocks) at time t.

For a building in damage state i, an earthquake which results in the building suffering further damage and going to damage state j, where $j > i$, is considered as a loss event. Thus, if the intensity function of earthquakes (either mainshock or aftershock) is denoted by $\mu(t)$, then the loss events for a building in state i can be modeled as a Poisson process with intensity function $\lambda_i(t)$, where:

$$\lambda_i(t) = \mu(t)\sum_{j>i} Q_{ij} = \mu(t)(1 - Q_{ii}) \tag{2}$$

In order to quantify the financial losses due to the occurrence of a loss event, the total transition and disruption losses need to be taken into consideration and appropriately discounted back to the present value. The disruption loss is highly dependent on the inter-arrival times between events (either mainshocks or aftershocks). For this initial simplified procedure, we assume that l_{ij} includes an expected disruption loss due to transition to damage state j. Also, the simplified Poisson model assumes that the building is immediately "re-built" back to its original damage state after the loss event. This might not be realistic in the aftershock environment where there might be incremental damage to the building due to the occurrences of aftershocks and where there might not be sufficient time for repair back to its original state. Such assumptions will be relaxed in the Markov process framework to follow.

Based on the damage states of the building, given a loss event, we can form a transition matrix $\mathbf{P}$ as defined in Eqs. (3)–(5).

$$\mathbf{P} = \begin{pmatrix} 0 & P_{12} & \cdots & P_{1n} \\ \vdots & \ddots & 0 & P_{(n-1)n} \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \tag{3}$$

$$P_{ij} = 0, \quad i \geqslant j \tag{4}$$

$$P_{ij} = \frac{\mu(t)Q_{ij}}{\lambda_i(t)} = \frac{Q_{ij}}{1 - Q_{ii}}, \quad i < j \tag{5}$$

$\mathbf{P}$ is a stationary transition matrix, i.e., it does not change with time. The values of the lower triangular portion and the diagonals of the matrix $\mathbf{P}$ are all equal to zero because we are only considering loss events, i.e., earthquakes that result in transitions to more severe damage states.

We denote $L(i)$ as the random variable of the financial losses given the occurrence of a loss event for a building in damage state i. The expected value of $L(i)$ can be calculated as $E[L(i)] = \sum_{j>i} P_{ij} l_{ij}$. In the formulation to follow, we consider the Poisson process of loss events with intensity function $\lambda_i(t)$ given in Eq. (2) for a building in damage state i. We suppress the dependence on the original damage state of the building (i) for $L(i)$ such that we represent $L(i)$ as L. We use the notations below for a building originally in damage state i, where capital letters denote random variables. We consider financial losses for both mainshocks and aftershocks.