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Cost–benefit analysis under uncertainty — A note on Weitzman's dismal theorem[☆,☆☆]

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ABSTRACT

Weitzman's (2009) famous dismal theorem argues that "fat tails" in the distribution of warming may pose problems for cost–benefit analysis as it may imply that society might be willing to exchange today's consumption for future consumption at an infinite rate. His analysis is based on the stochastic discount factor. We show that in situations in which the stochastic discount factor is applicable, it is optimal for society to devote only a finite amount of resources to protect against climate change. For general assumptions on the investment returns, cost–benefit analysis must consider the joint distribution of the marginal utility of future consumption and marginal returns to investment in the different future states of nature. We explore the range of situations under which challenges for applying cost–benefit analysis under uncertainty remain.

1. Introduction

Weitzman (2009) laid out a dismal theorem which shows a potential problem for applying cost–benefit analysis in the realm of large structural uncertainty. Using the example of climate change, he shows that the rate at which society would be willing to exchange today's consumption for future consumption might very well be infinite. The implication is that climate change may be a situation in which society should be willing to devote all of its resources to protect against future climate change. The conclusions from the dismal theorem have received an intense discussion (e.g., Karp, 2009; Nordhaus, 2009, 2011).

In this note, we argue that Weitzman's derivation is missing a crucial term. This term represents the technological and policy options available for addressing climate change, an essential element of models for cost–benefit analysis. Since both future income and the future returns to investment may be subject to the same underlying uncertainties, any decision criterion must be based on their joint distribution. Thus, cost–benefit analysis cannot generally be based on the stochastic discount factor which Weitzman takes as the basis for his arguments. We reformulate the model to explicitly account for investment options and find that it rarely implies that society should "invest everything". We show that the infinity result can still occur, although under narrower conditions than Weitzman originally claimed.[1]

2. The basic point

Following Weitzman we consider a two-period model with utility $U(C)$ where C is consumption. Let C_t be consumption in period $t \in \{0,1\}$. C_1 is random from the point of view of a period 0 decision-maker and depends on first period decisions. The discount factor is β. Society's objective function is given by:

$$U(C_0) + \beta E[U(C_1)]. \tag{1}$$

For the moment, we ignore the question of whether the expectation $E[U(C_1)]$ exists. In the text below we provide a more formal treatment of these elements. Weitzman argues that decisions about substitution between C_0 and C_1 in Eq. (1) depend on the stochastic discount factor:

$$\beta \frac{E\left[U'(C_1)\right]}{U'(C_0)}. \tag{2}$$

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[1] Note that we do not question Weitzman's claim that climate change is characterized by fat tails nor that the possibility of catastrophic outcomes from climate change is critically important for decisions about controlling greenhouse gas emissions.

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He describes this stochastic discount factor as "a kind of shadow price for discounting future costs and benefits in project analysis" (p. 7) and correctly derives conditions under which this term may be infinite: for utility functions of the form $U(C) = C^{1-\eta}/(1-\eta)$ and distributions of C_1 with fat tails, the ratio in Eq. (2) will be infinite because $E[C^{-\eta}] = \infty$ for $\eta > 0$ (normalizing $C_0 = 1$). Weitzman concludes that this poses a challenge to applying cost–benefit analysis as it essentially suggests that society should be willing to transfer all of its period 0 resources to period 1.

However, Eq. (2) does not reflect how today's actions affect future utility and therefore does not represent a relevant expression for cost–benefit analysis. While Eq. (2) describes the rate at which an agent would be willing to give up current consumption for one additional (sure) unit of future consumption, such a technology of generating a sure unit in the future may not be available.

The correct more general expression for trading off today's consumption and (uncertain) future consumption is:

$$\beta\frac{E\left[U'(C_1)\frac{dC_1}{dC_0}\right]}{U'(C_0)}. \tag{3}$$

In words, the expression in Eq. (2) is missing the term dC_1/dC_0. This is a "technology" term, which captures the ways in which current investments (i.e., foregone consumption in period 0) pay off in the future. This technology is a crucial element for decisions about abating climate change.

Expression (3) shows that marginal utility and marginal returns from investment in specific states of nature in general cannot be separated. We therefore immediately obtain the following result.[2]

Proposition 1. *The use of the expected marginal rate of substitution between today's and future income is generally not a valid indicator for cost–benefit analysis. It only can be used, if marginal investment returns are safe.*

Proposition 1 implies that results regarding Weitzman's expression (2), in particular the fact that it may equal infinity, may not extend to a proper cost–benefit analysis.

In the next section we analyze more rigorously the maximization problem corresponding to Eq. (1) under different kinds of technologies. The analysis proceeds using the correct expression in Eq. (3).

3. Cost–benefit analysis under uncertainty: implications of different abatement technologies

To model this relationship between investment today and future payoffs, i.e., dC_1/dC_0, we consider an action $p \in [0,1]$ which is taken in period 0 and yields current net consumption $C_0 = 1 - p$ and future consumption $C_1 = C(p,s)$, where s is a random variable representing the future state of nature. This notation is completely general and merely makes explicit the policy options that are implicitly assumed in Weitzman. We define $C'(p,s) = \partial C(p,s)/\partial p = -dC_1/dC_0$ and adopt innocuous regularity assumptions (concavity).

We maximize Eq. (1) by choosing p. An interior solution to this investment problem requires the first-order condition

$$H(p) \equiv \beta\frac{E\left[U'(C(p,s))C'(p,s)\right]}{U'(1-p)} = 1. \tag{4}$$

The left-hand-side expression is precisely the same as Eq. (3). A corner solution results either if $H(0) < 1$, which implies optimal $p^* = 0$, or if $H(p) > 1$ for all $p \in [0, 1)$, which yields $p^* = 1$.

Our purpose in the following analysis is not to argue for a particular technology, but to show how assumptions about technologies $C(p,s)$ translate into decisions using the expression $H(p)$.

Case 1. We consider first the case in which a technology exists which generates a *safe* marginal return to investment, i.e. $C'(p,s) = \hat{C}(p)$ for all s. This is the situation that Weitzman appears to consider, since he refers to his model as describing the tradeoff between present consumption and "one extra sure unit of consumption in the future" (p. 6–7).[3] We rewrite this as $C(p,s) = \hat{C}(p) + R(s)$, where we assume without loss of generality that $\hat{C}(0) = 0$. Since consumption levels are assumed to be non-negative, we also assume $R(s) \geq 0$. Note that the dismal theorem is concerned with situations where the future consumption is close to zero.

Under such an assumption of a safe marginal return on investment, an interior solution requires

$$\beta\frac{E\left[U'\left(\hat{C}(p)+R(s)\right)\right]}{U'(1-p)} = \frac{1}{\hat{C}'(p)}. \tag{5}$$

Here, the left hand side describes the marginal rate at which the agent would be willing to give up present consumption to obtain one sure unit of consumption in the future, i.e. the indicator used by Weitzman. The right hand side gives the marginal costs of such a safe investment.

We obtain the following proposition:

Proposition 2. *If there is an investment option which generates a safe return on today's investment, it cannot be optimal to give up all current wealth in order to mitigate climate change. The resulting optimal expected marginal rate of substitution between current and future consumption is finite.*

Proof. We first note that $\lim_{p\to 1} U'(1-p) = \infty$ and $U'\left(\hat{C}(p)+R(s)\right) < U'\left(\hat{C}(p)\right) < \infty$ for $\hat{C}(p) > 0$. Therefore

$$\beta\frac{E\left[U'\left(\hat{C}(p)+R(s)\right)\right]}{U'(1-p)} < \beta\frac{E\left[U'\left(\hat{C}(p)\right)\right]}{U'(1-p)} \to 0$$

for $p \to 1$. With condition (5), this implies that a corner solution at $p = 1$ cannot be optimal if a safe investment option exists. In any interior solution, however, the right hand side of Eq. (5) is finite. The same holds for a corner solution at $p = 0$ which results if the first-order condition satisfies

$$\beta\frac{E\left[U'\left(\hat{C}(0)+R(s)\right)\right]}{U'(1)} < \frac{1}{\hat{C}'(0)}. \tag{6}$$

□

Proposition 2 shows that — as long as a safe investment option exists — a society would neither give up all current consumption, nor would the expected marginal rate of substitution between consumption today and in the future be infinite.

[2] Assume that $E[U'(C_1(C_0))C_1'(C_0)] = E[U'(C_1(C_0))]E[C_1'(C_0)]$ for all C_0, i.e. assume that $U'(C_1(C_0))$ and $C_1'(C_0)$ are independent random variables. Then, differentiation implies $E[U''(C_1(C_0))C_1'(C_0)^2] = E[U''(C_1(C_0)C_1'(C_0))]E[C_1'(C_0)]$. This implies either (i) $U''(C_1) = 0$ in which case $U'(C_1)$ is constant and considering $E[U'(C_1)]$ is meaningless, or (ii) $E[C_1'(C_0)^2] = E[C_1'(C_0)]^2$ which implies that $C_1'(C_0)$ does not depend on the state of nature, i.e. that returns from investment are safe.

[3] Weitzman is not always clear about whether his canonical case refers to certain or uncertain returns to investment. He states that throughout the paper he uses "the price of a future sure unit of consumption…as the single most useful overall indicator of the present cost of future uncertainty." (p. 6) Later (p. 11), however, he states that "if a particular type of idiosyncratic uncertainty affects only a small part of society's overall portfolio of assets, exposure is naturally limited to that specific component and bad-tail fatness is not such a paramount concern", which suggests that he may have uncertain future returns in mind.