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Physica A 324 (2003) 81–88
PHYSICA A
www.elsevier.com/locate/physa

Risk and utility in portfolio optimization

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Abstract

Modern portfolio theory (MPT) addresses the problem of determining the optimum allocation of investment resources among a set of candidate assets. In the original mean-variance approach of Markowitz, volatility is taken as a proxy for risk, conflating uncertainty with risk. There have been many subsequent attempts to alleviate that weakness which, typically, combine utility and risk. We present here a modification of MPT based on the inclusion of separate risk and utility criteria. We define risk as the probability of failure to meet a pre-established investment goal. We define utility as the expectation of a utility function with positive and decreasing marginal value as a function of yield. The emphasis throughout is on long investment horizons for which risk-free assets do not exist. Analytic results are presented for a Gaussian probability distribution. Risk-utility relations are explored via empirical stock-price data, and an illustrative portfolio is optimized using the empirical data.

PACS: 01.30.Cc; 02.50.−r; 02.50.Ey; 89.90.+n

Keywords: Portfolio; Risk utility; Mean-variance; Finance

1. Introduction

Two of the main pillars of mathematical finance are modern portfolio theory (MPT) and the Capital Asset Pricing Model (CAPM). The seminal work on MPT is attributed to Markowitz who presented his mean-variance approach to asset allocation in 1952 [1]. It was soon amplified by Sharpe in 1964 [2] and by Lintner in 1965 [3] with the introduction of the concept of the capital market line and subsequent development of the CAPM. MPT permeates the teaching and practice of classical financial theory.

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0378-4371/03/$ - see front matter
doi:10.1016/S0378-4371(02)01957-X

Substantial portions of most textbooks on finance are devoted to it and its implications. Its influence has been profound.

The notion that portfolio volatility, the square root of the variance of the portfolio yield, is an adequate proxy for risk is fundamental to MPT. Similarly, the notion that there exists at least one risk-free asset is fundamental to the construction of the capital market line and the formulation of the CAPM. In the present paper, we discuss issues surrounding both of these notions and, abandoning them, introduce a novel method of portfolio optimization. The notion that variance measures risk is now viewed as a weak compromise with economic reality. Variance measures uncertainty, and there are circumstances of interest in which great uncertainty implies little risk. Similarly, supposing that there are risk-free assets or, more precisely, assets with unvarying yield is a poor approximation, particularly for long-time horizons.

There have been attempts to develop MPT with alternative definitions of risk, including a semi-variance, RMS loss, average downside risk, value at risk (VAR) and others [4–7] but to our knowledge, none is based on the classic notion that the probability of failure to meet a preset goal is the proper quantitative measure of risk or on the elimination of the notion of a risk-free asset.

In the following sections we give a brief introduction of MPT with critiques of each of the above two fundamental notions. We show that the probability of success can be interpreted as an expected utility that is deficient in some desirable features. We construct an additional utility with the desired properties and include it in the portfolio optimization. We discuss how to define a real portfolio optimization problem using historical data and report the result of our risk and utility evaluation using the daily closing prices for 13,000 stocks listed on the NYSE and NASDAQ during the period 1977–1996. We conclude by presenting the results of our optimization for a portfolio drawn from a subset of low risk, high utility stocks and discuss the implications of our main findings.

2. Modern portfolio theory

The asset allocation problem is one of the fundamental concerns of financial theory. It can be phrased as a question: What is the optimal allocation of funds F among a set of assets $\{A_i\}$ for a predetermined level of risk? Phrased in this way it leaves unspecified the meaning of optimality and of risk. Modern portfolio theory offers a two step answer via a particular specification of optimality and risk. The first step was taken in 1952 with the introduction of the mean-variance approach of Markowitz [1]. By equating risk with variance, Markowitz derived an efficient frontier of portfolios which maximize return for given risk and opened the door to further advances in this theoretical framework.

The addition of a risk-free asset by Sharpe [2] and Lintner [3] in the mid 1960s led to the capital market line and the CAPM. They supposed that there exists a risk-free asset A_0, whose yield, Y_0, did not fluctuate. The line drawn from Y_0 tangent to the efficient frontier is then the locus of yields of all optimal portfolios which can be constructed by adding the risk-free asset to holdings drawn from the