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Portfolio optimization with stochastic dominance constraints ☆

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Abstract

We consider the problem of constructing a portfolio of finitely many assets whose return rates are described by a discrete joint distribution. We propose a new portfolio optimization model involving stochastic dominance constraints on the portfolio return rate. We develop optimality and duality theory for these models. We construct equivalent optimization models with utility functions. Numerical illustration is provided.

JEL classification: C44; C61; G11

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1. Introduction

The problem of optimizing a portfolio of finitely many assets is a classical problem in theoretical and computational finance. Since the seminal work of Markowitz

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(1952, 1959, 1987) it is generally agreed that portfolio performance should be measured in two distinct dimensions: the *mean* describing the expected return rate, and the *risk* which measures the uncertainty of the return rate. In the mean–risk approach, we select from the universe of all possible portfolios those that are *efficient*: for a given value of the mean they minimize the risk or, equivalently, for a given value of risk they maximize the mean. This approach allows one to formulate the problem as a parametric optimization problem, and it facilitates the trade-off analysis between mean and risk.

Another theoretical approach to the portfolio selection problem is that of *stochastic dominance* (see Mosler and Scarsini, 1991; Whitmore and Findlay, 1978). The concept of *stochastic dominance* is related to models of risk-averse preferences (Fishburn, 1964). It originated from the theory of majorization (Hardy et al., 1934; Marshall and Olkin, 1979) for the discrete case, was later extended to general distributions (Quirk and Saposnik, 1962; Hadar and Russell, 1969; Rothschild and Stiglitz, 1969), and is now widely used in economics and finance.

The usual (first order) definition of stochastic dominance gives a partial order in the space of real random variables. More important from the portfolio point of view is the notion of second-order dominance, which is also defined as a partial order. It is equivalent to this statement: a random variable R dominates the random variable Y if $\mathbb{E}[u(R)] \geqslant \mathbb{E}[u(Y)]$ for all non-decreasing concave functions $u(\cdot)$ for which these expected values are finite. Thus, no risk-averse decision maker will prefer a portfolio with return rate Y over a portfolio with return rate R.

In our earlier publications (Dentcheva and Ruszczyński, 2003a,b,c, 2004a,c) we have introduced a new stochastic optimization model with stochastic dominance constraints. In this paper we show how this theory can be used for risk-averse portfolio optimization. We add to the portfolio problem the condition that the portfolio return rate stochastically dominates a benchmark return rate, for example, the return rate of an index. We identify concave non-decreasing utility functions which correspond to dominance constraints. Maximizing the expected return rate modified by these utility functions, guarantees that the optimal portfolio return rate will dominate the given benchmark return rate.

This approach has a fundamental advantage over mean–risk models and utility function models. All data for our model are readily available. In mean–risk models the choice of the risk measure has an arbitrary character, and it is difficult to argue for one measure against another. Also the choice of the weight of risk in the mean–risk model is somewhat arbitrary, and one has to consider entire families of risk measures and risk weights, for decision support. Similarly, optimization of expected utility requires the form of the utility function to be specified. Utility functions of decision makers are very difficult to elicit. The situation is even more complicated when there is a group of decision-makers who have to come to a consensus. Our model avoids all these difficulties, by requiring that a benchmark random outcome, considered reasonable, be specified. This may be, for example, the market index.

Our analysis, departing from the benchmark outcome, generates the utility function of the decision maker. It is implicitly defined by the benchmark used, and by the problem under consideration.