Available online at www.sciencedirect.com
ScienceDirect
Information Sciences 178 (2008) 1734–1755
INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL
www.elsevier.com/locate/ins

Asset portfolio optimization using fuzzy mathematical programming

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Received 28 January 2007; received in revised form 31 July 2007; accepted 26 October 2007

Abstract

By morphing mean–variance optimization (MVO) portfolio model into semi-absolute deviation (SAD) model, we apply multi criteria decision making (MCDM) via fuzzy mathematical programming to develop comprehensive models of asset portfolio optimization (APO) for the investors' pursuing either of the aggressive or conservative strategies.

Keywords: Fuzzy sets; Portfolio selection; Fuzzy mathematical programming; Risk; Semi-absolute deviation; Multi criteria decision making

1. Introduction

The portfolio selection problem deals with how to form a satisfying portfolio. It is difficult to decide which assets should be selected because of the uncertainty on their returns. The main objective in a portfolio selection problem is to obtain optimal proportions of the stock for creating a portfolio which respects the investor's preferences assuming that the investors' wish to strike a balance between maximizing the return and minimizing the risk of their investment.

Modern portfolio analysis started from pioneering research work of Markowitz [24]. The portfolio selection model was first formulated by Markowitz, which is called mean–variance model. Based on this model, absolute deviation portfolio optimization [13] and semi-absolute deviation portfolio optimization models [23] were proposed.

In the aforementioned portfolio selection models, decision maker must confirm that all of the information available or needed is brought to bear on the problem or issues at hand. However, identifying all relevant information for a decision does not mean that the decision makers have complete information; in most instances, information is incomplete. Decisions must be made with limited information because decision

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0020-0255/$ - see front matter
doi:10.1016/j.ins.2007.10.025

makers do not have full knowledge of the problem they face and generally cannot even determine a reasonable probability for alternative outcomes; thus they must make their decisions under conditions of uncertainty. Though probability theory is one of the main techniques used for analyzing uncertainty in finance, the financial market is also affected by several non-probabilistic factors such as vagueness and ambiguity. Decision makers are commonly provided with information which is characterized by linguistic descriptions such as high risk, low profit, high interest rate, etc. [27]. With the introduction of fuzzy set theory by Zadeh [36], it was realized that imperfect knowledge of the returns on the assets and the uncertainty involved in the behaviour of financial markets may be captured by means of fuzzy quantities and/or fuzzy constraints. A review of literature on application of fuzzy set theory in the problem of portfolio selection suggests a variety of approaches in doing so. Some authors have used possibility distributions to model the uncertainty on returns, while other authors have studied the portfolio selection problem using fuzzy formulations. For example, in Watada [34] the vague goals for expected return and risk are introduced to consider fuzzy portfolio selection problem. In León et al. [18] a fuzzy approach is proposed to describe soft constraints and repair infeasibility in portfolio optimization problem. In Tanaka and Guo [29], Tanaka et al. [30] the possibility theory is applied to handle uncertainty and solve portfolio optimization problem. Lai et al. [15], Wang and Zhu [32] and Giove et al. [10] used linear interval programming models for portfolio selection. Carlsson et al. [5] introduced a possibilistic approach for selecting portfolios with the highest utility value under the assumption that the returns of assets are trapezoidal fuzzy numbers. Wang et al. [33] and Zhang and Wang [37] discussed the general weighted possibilistic portfolio selection problems. Lacagnina and Pecorella [14] developed a multistage stochastic soft constraints fuzzy program with recourse in order to capture both uncertainty and imprecision as well as to solve a portfolio management problem. Lin et al. [21] proposed a systematic approach by incorporating fuzzy set theory in conjunction with portfolio matrices to assist managers in reaching a better understanding of the overall competitiveness of their business portfolios. Huang [11] provided two portfolio selection models with fuzzy returns by criteria of chance represented by credibility measure. In another paper, Huang [12] proposed two models for portfolio selection in which the security returns are stochastic variables with fuzzy information. Fei [9] studied the optimal consumption and portfolio choice with ambiguity and anticipation. Bilbao et al. [4] applied fuzzy compromise programming for portfolio selection problem. Ammar [1] solved the fuzzy portfolio optimization problem as a convex quadratic programming problem and provided an acceptable solution for it. Zhang et al. [38] proposed two kinds of portfolio selection models based on lower and upper possibilistic means and possibilistic variances, respectively, and introduced the notions of lower and upper possibilistic efficient portfolios.

In the above cited works, expected return and risk are two fundamental factors which investors' consider. It is often found in portfolio selection that not all the relevant information for an investment decision can be captured in terms of explicit return and risk. By considering additional and/or alternative decision criteria, a portfolio that is dominated with respect to expected return and risk may make up for the deficit in these two criteria by a very good performance on one or several other criteria and thus be non-dominated in a multicriteria setting. As a result, portfolio selection models that consider more criteria than the standard expected return and variance objectives of the Markowitz model have become popular. Arenas et al. [2] proposed a model that consider three criteria viz., return, risk and liquidity. Ehrgott et al. [7] took into account five criteria (short and long term return, dividend, ranking and risk) and used multi criteria decision making (MCDM) approach to solve the portfolio selection problem. Fang et al. [8] proposed a portfolio rebalancing model with transaction costs based on fuzzy decision theory considering three criteria (return, risk and liquidity).

In this paper, the focus of the research is to incorporate fuzzy set theory into a semi-absolute deviation portfolio selection model for investors' taking into account five criteria: short term return, long term return, dividend, risk and liquidity. The expected return as used in the Markowitz model has been broken into the criteria: short term return, long term return and annual dividend in order to improve the possibilities of an investor to articulate subjective preferences. In the proposed model, for a given return level, the investor penalizes the negative semi-absolute deviation that is defined as a risk. From computational point of view, the semi-absolute deviation halves the number of required constraints with respect to the absolute deviation. Liquidity has been measured as the degree of probability involved in the conversion of an investment into cash without any significant loss in value. Additionally, we consider constraints like the minimal and maximal fraction of