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Random matrix theory filters in portfolio optimisation: A stability and risk assessment

J. Daly*, M. Crane, H.J. Ruskin

Dublin City University, Glasnevin, Dublin 9, Ireland

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Abstract

Random matrix theory (RMT) filters, applied to covariance matrices of financial returns, have recently been shown to offer improvements to the optimisation of stock portfolios. This paper studies the effect of three RMT filters on the realised portfolio risk, and on the stability of the filtered covariance matrix, using *bootstrap* analysis and *out-of-sample* testing.

We propose an extension to an existing RMT filter, (based on Krzanowski stability), which is observed to reduce risk and increase stability, when compared to other RMT filters tested. We also study a scheme for filtering the covariance matrix directly, as opposed to the standard method of filtering correlation, where the latter is found to lower the realised risk, on average, by up to 6.7%.

We consider both equally and exponentially weighted covariance matrices in our analysis, and observe that the overall best method *out-of-sample* was that of the exponentially weighted covariance, with our Krzanowski stability-based filter applied to the correlation matrix. We also find that the optimal out-of-sample decay factors, for both filtered and unfiltered forecasts, were higher than those suggested by Riskmetrics [J.P. Morgan, Reuters, Riskmetrics technical document, Technical Report, 1996. http://www.riskmetrics.com/techdoc.html], with those for the latter approaching a value of $\alpha = 1$.

In conclusion, RMT filtering reduced the realised risk, on average, and in the majority of cases when tested out-of-sample, but increased the realised risk on a marked number of individual days–in some cases more than doubling it.

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1. Introduction

Markowitz portfolio theory [2], an intrinsic part of modern financial analysis, relies on the covariance matrix of returns and this can be difficult to estimate. For example, for a time series of length T, a portfolio of N assets requires $(N^2 + N)/2$ covariances to be estimated from NT returns. This results in estimation noise, since the availability of historical information is limited. Moreover, it is commonly accepted that financial covariances are not fixed over time (e.g. Refs. [1,3,4]) and thus older historical data, even if available, can lead to cumulative noise effects.

* Corresponding author.
E-mail addresses: jdaly@computing.dcu.ie (J. Daly), mcrane@computing.dcu.ie (M. Crane), hruskin@computing.dcu.ie (H.J. Ruskin).

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Random matrix theory (RMT), first developed by authors such as Dyson and Mehta [5–8], to explain the energy levels of complex nuclei [9], has recently been applied to noise filtering in financial time series, particularly in large dimensional systems such as stock markets, by several authors including Plerou et al. [9–13] and Laloux et al. [14,15]. Both groups have analysed the US stock markets and have found that the eigenvalues of the correlation matrix of returns are consistent with those calculated using random returns, with the exception of a few large eigenvalues. Moreover, their findings indicated that these large eigenvalues, which do not conform to random returns, had eigenvectors that were more stable over time. Of particular interest was the demonstration [9,15] that filtering techniques, based on RMT, could be beneficial in portfolio optimisation, both reducing the realised risk of optimised portfolios, and improving the forecast of this realised risk.

More recently, Pafka et al. [16] extended RMT to provide Riskmetrics type [1] covariance forecasts. Riskmetrics, dating from the 1990s, and considered a benchmark in risk management [16], uses an exponential weighting to model the heteroskedasticity of financial returns. Pafka et al. [16] showed that RMT-based eigenvalue filters can improve the optimisation of minimum risk portfolios, generated using exponentially weighted forecasts. However, these authors found that the decay factors which produced the least risky portfolios were higher than the range suggested by Riskmetrics and further concluded that unfiltered Riskmetrics-recommended forecasts were unsuitable for their portfolio optimisation problem. A recent paper by Sharifi et al. [17], using equally weighted, high frequency returns for estimating covariances, proposed an alternative eigenvalue-filtering method, based on a principal components technique developed by Krzanowski [18] for measuring the stability of eigenvectors, in relation to small perturbations in the corresponding eigenvalues. Sharifi et al. [17] concluded that filtering correlation matrices according to the method outlined in Laloux et al. [15] had a negative effect on this stability.

Our objectives in this article are: (i) to present a computationally efficient method for calculating the maximum eigenvalue of an exponentially weighted random matrix; (ii) to study the behaviour of the stability-based filter [17] for daily data and for exponentially weighted covariance; (iii) to explore the possibility of filtering the covariance matrix directly (as opposed to the standard method of filtering correlation); and (iv) to compare three available RMT filters using bootstrapping and out-of-sample testing. The paper is organised as follows. In Section 2, we review the theoretical background for the three RMT filters, Section 3 contains the in-sample analysis of the filters from a stability and risk reduction perspective, and in Section 4 we present results of the out-of-sample test on the effectiveness of the filters in reducing risk. In the Appendix, we describe the filtering methods of Laloux et al. [15] and Plerou et al. [9].

2. Background

2.1. Random matrix theory and historical covariance

As described by Laloux et al. [14], Plerou et al. [9], Sharifi et al. [17] and others, in the context of correlation matrices of financial returns, if **R** is any matrix defined by

$$\mathbf{R} = \frac{1}{T}\mathbf{A}\mathbf{A}' \tag{1}$$

where **A** is an $N \times T$ matrix whose elements are i.i.d.[1] random variables with a zero mean, then it has been shown [19] that, in the limit $N \to \infty$, $T \to \infty$ such that $Q = T/N \geq 1$ is fixed, the distribution $P(\lambda)$ of the eigenvalues of **R** is self-averaging, and is given by

$$P(\lambda) = \begin{cases} \dfrac{Q}{2\pi\sigma^2}\dfrac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} & \text{if } \lambda_- \leq \lambda \leq \lambda_+ \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where σ^2 is the variance of the elements of **A** and

$$\lambda_\pm = \sigma^2\left(1 + 1/Q \pm 2\sqrt{1/Q}\right). \tag{3}$$

[1] i.i.d. ≡ independent and identically distributed.