



Multi-period portfolio optimization with linear control policies[☆]

Giuseppe Carlo Calafiore

Dipartimento di Automatica e Informatica, Politecnico di Torino, Italy

ARTICLE INFO

Article history:

Received 28 March 2007
 Received in revised form
 2 October 2007
 Accepted 3 February 2008
 Available online 13 May 2008

Keywords:

Risk management
 Finance
 Dynamic portfolio allocation
 Multi-stage decisions
 Convex optimization

ABSTRACT

This paper is concerned with multi-period sequential decision problems for financial asset allocation. A model is proposed in which periodic optimal portfolio adjustments are determined with the objective of minimizing a cumulative risk measure over the investment horizon, while satisfying portfolio diversity constraints at each period and achieving or exceeding a desired terminal expected wealth target. The proposed solution approach is based on a specific affine parameterization of the recourse policy, which allows us to obtain a sub-optimal but exact and explicit problem formulation in terms of a convex quadratic program.

In contrast to the mainstream stochastic programming approach to multi-period optimization, which has the drawback of being computationally intractable, the proposed setup leads to optimization problems that can be solved efficiently with currently available convex quadratic programming solvers, enabling the user to effectively attack multi-stage decision problems with many securities and periods.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

The fundamental goal of portfolio theory is to help the investor allocating money among different financial securities in an “optimal” way. In the classical Markowitz framework, (Markowitz, 1991), the selection is guided by a quantitative criterion that considers a tradeoff between the return of an investment and its associated risk. Specifically, in the Markowitz approach, each asset is described by means of its return over a fixed period of time (e.g. one month), and the vector of asset returns is assumed to be random, with known expectation and covariance matrix. An optimal portfolio of assets is hence selected, for instance, by minimizing the investment risk (as expressed by the portfolio variance, or “volatility”) subject to a given lower bound on the expected return at the end of the period. From the computational side, this classical paradigm results in a quadratic programming problem, which may be efficiently solved numerically on a computer.

However, a drawback of this basic approach is that it is tuned to a single period, and it can therefore provide short-sighted strategies of investment, if applied repeatedly over many subsequent periods. To overcome this issue, one may formulate from the beginning the allocation problem over an horizon composed of multiple periods ($T \geq 1$ periods), with the goal of minimizing the total risk over the investment path, while satisfying

constraints on the portfolio composition and on desired expected return at all the intermediate stages.

Seminal contributions to multi-stage decisions in finance have been given in Merton (1971, 1973), where an approach based on continuous-time dynamic programming is proposed. This paradigm is still in use – see for instance (Ait-Sahalia & Brandt, 2001; Brennan, Schwartz, & Lagnado, 1997; Lynch & Balduzzi, 2000). Note, however, that the dynamic programming approach is impractical for actual numerical implementation, due to the “curse of dimensionality.” Indeed, Brandt (1999) and Brennan et al. (1997) report that incorporating more than a few state variables in the dynamic programming formulation leads to unworkable problem size (and probably for this reason most of the multi-period models encountered in the literature are actually two-periods models with only a few securities). Analytical or reduced complexity solutions can be obtained only in very special cases. For instance, a closed form solution in continuous-time is proposed in Zhou and Li (2000) when no transaction costs are considered and no constraints are imposed on the portfolio composition. On a similar note, a mean-variance discrete-time problem is reduced to a control problem with only one state variable in Infanger (2006), under the hypotheses of no transaction costs, no composition constraints and serially independent returns. It should hence be remarked that the presence of constraints on portfolio composition and/or of transaction costs makes the problem harder from the computational viewpoint.

The mainstream computational model to solve recursive decision problems in the presence of uncertainty is currently provided by multi-stage stochastic programming, see e.g. Birge (1997), Birge and Louveaux (1997), Gulpinar, Rustem, and Settergren (2002), and Ruszczyński and Shapiro (2003) and the many references

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Editor Berc Rüstern.

E-mail address: giuseppe.calafiore@polito.it.

therein. However, while stochastic programming provides a conceptually sound framework for posing multi-stage decision problems, from the computational side it results to be impervious to exact and efficient numerical solution, (Shapiro, 2005). The key difficulty in the stochastic programming formulation comes from the fact that the stage decisions are actually conditional decision rules, or “policies” that define which action should be taken in response to past outcomes. To model the conditional nature of the problem in some “tractable” way, a discretization of the decision space is typically introduced by constructing a “scenario tree,” and this scenario tree may grow exponentially if an accurate and representative discretization is needed, see e.g. Ermoliev and Wets (1988). On the other hand, if branching is kept low in the scenario tree, the resulting discretization cannot be guaranteed to be a reliable representation of reality. These computational difficulties are witnessed by the fact that most multi-period problems discussed in the literature deal with few securities over only two or three periods.

In the specific context of financial allocation, a classical stochastic programming method based on Benders decomposition is proposed in Dantzig and Infanger (1993), and techniques for construction of scenario trees are discussed for instance in Gulpinar, Rustem, and Settergren (2004), Pflug (2001) and Ziemba and Mulvey (1998). A possibly improved approach that uses scenario trees together with simulated paths has been recently studied in Hibiki (2006). Also, in Rustem and Gulpinar (2007), the authors extend the multi-period mean-variance model to deal with rival uncertainty scenarios, and propose a worst-case decision approach which uses a min-max technique in synergy with a stochastic optimization algorithm based on scenario trees. Scenario-based stochastic programming models aiming at maximizing expected portfolio value while taking into account cost variability have been recently proposed also in Pinar (2007) and Takriti and Ahmed (2004). A survey with theoretical analysis of multiperiod models based on scenario trees is provided in Steinbach (2001).

In this paper, we propose a different route to multi-stage portfolio allocation, which prescind from the use of decision trees or sample paths, and which leads to explicit convex constrained quadratic programming models that can be solved globally and efficiently.

We achieve this goal by considering recourse actions that are prescribed by policies with fixed structure. In particular, we shall consider recourse actions that are affine functions of the past periods returns, where the coefficients of these functions become the decision variables of the problem. While with this position we lose some generality, since the control policy is now restricted to the affine functions class, we also gain decisive advantages. First, we show that it is possible to express explicitly the expected value and variance of the portfolio at any stage, as a function of the decision variables. Furthermore, the optimization objective and constraints result to be convex quadratic functions of these variables, and therefore the optimal strategy, under the affine recourse hypothesis, can be found *exactly* and *numerically efficiently* by means of standard codes for convex quadratic programming. Second, the optimal recourse parameters returned by the algorithm have a simple and insightful interpretation as nominal actions and market reaction sensitivities, as further discussed in Section 5.

The idea of using affine recourse has been inspired by a similar “adjustable variables” technique recently proposed in Ben-Tal, Goryashko, Guslitzer, and Nemirovski (2004) in the context of robust optimization (i.e. optimization problems where the data is subject to deterministic unknown-but-bounded uncertainties), where the authors impose an affine dependence of variables on the data, so that variables can “tune themselves to varying data”. Adjustable variables have been proposed in the context of model

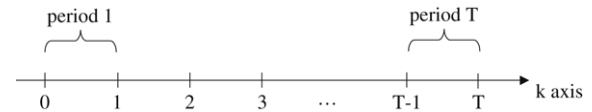


Fig. 1. Time axis and periods for multi-period allocation.

predictive control in Löfberg (2003), and, more generally, affine recourse is reminiscent of the classical linear feedback laws used for control of dynamical systems. A similar idea is also employed in dynamic programming for approximating the optimal cost-to-go function by fitting a parameterized function approximator, see for instance Bertsekas and Tsitsiklis (1996); Farias and Van Roy (2006).

This paper is organized as follows. Preliminary concepts are given in Section 2. Section 2.1 describes the recursive equations for the portfolio dynamics, and Section 3 presents the basic open-loop mean-variance optimization setup. The key Section 4 introduces the recourse model into the dynamic decision problem, and states a convexity result for affinely parameterized policies, in Lemma 1. An explicit multi-stage optimization model is developed in Section 5, under an additional assumption of independence of the market gains and a specific structure of the recourse rule. Numerical experiments are presented in Section 6, and conclusions are drawn in Section 7. Technical proofs are contained in the Appendix.

Notation. If $x \in \mathbb{R}^{n,1}$ is an n -dimensional vector, then $\text{diag}(x)$ denotes a diagonal matrix having the entries of x on the diagonal. A^\top denotes the transpose of matrix A . The operator \odot denotes the Hadamard (entry-wise) product of conformably sized matrices. For a random vector x taking values in $\mathbb{R}^{n,1}$, we denote with $E\{x\}$ the expected value of x , and with $\text{var}\{x\} \doteq E\{(x - E\{x\})(x - E\{x\})^\top\}$ the covariance matrix of x . We shall denote with an over bar the expectation of random quantities and with a tilde the centered quantities, i.e. the quantities with the expectation subtracted, that is $\bar{x} \doteq E\{x\}$, $\tilde{x} \doteq x - \bar{x}$.

2. Preliminaries

An investment instrument that can be bought and sold is here named an *asset*. In our discussion, an asset can be a single stock, cash, or an index representing the value of a class of securities. It is assumed that no dividends/coupons are paid, or if paid, they are immediately reinvested in the same asset.

Consider a universe of n assets $\{a_1, \dots, a_n\}$ and an investment timeline divided into T periods of equal duration Δ , as depicted in Fig. 1. Δ is a fixed time span, for instance $\Delta = 1$ month, or $\Delta = 1$ year.

We denote with $x_i(k)$ the Euro value of the portion of the investor’s total wealth invested in security a_i at time k . The vector having components $x_i(k)$, $i = 1, \dots, n$ is here named the *portfolio*:

$$x(k) \doteq \begin{bmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{bmatrix}.$$

The investor’s total wealth at time k is given by

$$w(k) = \sum_{i=1}^n x_i(k) = \mathbf{1}^\top x(k),$$

where $\mathbf{1}$ denotes a vector of ones of suitable dimension. At the end of each period, the investor has the opportunity of adjusting his/her investment, by rebalancing the portfolio composition. Specifically, we denote with $x^+(k)$ the portfolio composition just after the adjustment $u(k)$ occurred at time k :

$$x^+(k) \doteq x(k) + u(k),$$

متن کامل مقاله

دریافت فوری ←

ISIArticles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات