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Portfolio optimization of equity mutual funds with fuzzy return rates and risks

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ABSTRACT

Portfolio selection is an important issue for researchers and practitioners. Focusing on equity mutual funds, this paper proposes a basic portfolio selection model in which future return rates and future risks of mutual funds are represented by triangular fuzzy numbers. Firstly, a cluster analysis is proposed to categorize the huge amount of equity mutual funds into several groups based on four evaluation indices: rates of return, standard deviation, turnover rate, and Treynor index, in order to aid investors in making the investment decision. The fuzzy optimization model is proposed to determine the optimal investment proportion of each cluster. The portfolio optimization problem is developed in two ways: to maximize the future expected return subject to the given greatest future risk, and to minimize the future risk subject to a required lowest future expected return. The proposed approaches are demonstrated by Taiwan equity mutual funds.

1. Introduction

Financial investments are especially important for individual and business financial managers due to current the low interest rate. Among all outlets of investments, mutual funds are one of investors' favorite tools. Mutual funds have the characteristics of accumulation of risk and return, hence investors with a limited budget can also profit. The objective of managing mutual funds is to disperse investment risk to the smallest degree. From the data given by SITCA (http://www.sitca.org.tw/), the growth of Taiwan's mutual funds market is increasing very rapidly, and by the end of December 2006, the size of funds reached approximately NT 1967 billions dollars, with a total of 508 funds traded. Among different types of funds, equity mutual funds play an important role in the market share.

Portfolio selection, as originally articulated by Markowitz (1952, 1959), has been one of the important fields of research in modern finance. This theory examines the trade-off of risk and return in the "mean–variance" context. According to Markowitz's mean-variance model, given a specific rate of risk, one can derive the maximum investment return by maximizing the expected return of portfolios; or for a given specific return rate, one can derive the minimum investment risk by minimizing the variance of portfolio. The mean–variance method has been challenged and modified by many studies since it was first proposed decades ago. One other popular work is the market model, proposed by Sharpe (1966, 1967, 1970) and Litner (1965), which simplified Markowitz's model by ignoring the covariance between returns.

Conventional portfolio selection models have an assumption that the future condition of stock market can be accurately predicted by historical data, but no matter how accurate the past data is, this premise will not exist in real financial markets, due to the high volatility of market environments. Therefore, fuzzy set theory, proposed by Zadeh (1978), has become a helpful tool in handling the imprecise conditions and attributes of portfolio selection. This paper considers the uncertainty of future return rates and risk, and thus presents return rates and risk in fuzzy numbers, and solves the optimal asset allocation by fuzzy optimization. The result will provide a more reasonable investment decision that is more suitable for the imprecise financial environment.

Section 2 briefly reviews the background of portfolio selection approaches and performance evaluation techniques. In Section 3 we discuss the portfolio selection model and cluster analysis. We present the numerical results for Taiwanese equity mutual funds in Section 4. Some concluding remarks and comments are provided in final section.

2. Literature review

This section introduces the background to portfolio selection approaches, fund performance evaluation measurement techniques, and the application of fuzzy set theory to the portfolio selection problem. Markowitz (1952, 1959) proposed a mean–variance methodology for the portfolio problem which has become central to research activities in modern financial theory during the past four decades. The aim of the mean–variance model is to generate portfolios that have maximum return at the same level of risk (variance), or the minimum variances at the same level of

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desired return. The expected return and the variance of a portfolio are, respectively, given by

$$E(R_p) = \sum_{i=1}^{n} x_i E(R_i), \quad \text{and} \tag{1a}$$

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} x_i x_j \sigma_{ij}, \tag{1b}$$

where x_i denotes the proportion of ith assets in the portfolio p, $E(R_i)$ the expected return rates of ith assets, and σ_{ij} denotes the covariance between the return rates of the assets i and j. The Markowitz model requires a complex computational process, thus Sharpe (1966) proposed the single-index model (or market model), in which the returns of various securities are assumed to be related to each other only through common dependence upon a certain market index, not necessarily to specify the covariance between each pair of securities. Sharpe (1967, 1970) and Litner (1965) developed the equilibrium market model and capital asset pricing model (CAPM).

Since mutual funds have become a general investment tool, several indices or techniques to measure the performance of mutual funds have also been presented. Past performance of mutual funds has a central role in providing investors with a reference of how to allocate capital in the next investment period and over time. The number of indices measuring mutual fund performance has proliferated. Sharpe (1966) introduced a measure for the performance of mutual funds by considering both systematic and unsystematic risks (called total market risk) as the measurement of excess return. Treynor (1965) suggested that a portfolio's performance can be measured using the ratio of the mean risk premium to the systematic risk of the portfolio over the evaluation period rather than using total risk. Jensen (1968) proposed the Jensen Index as a more absolute performance measure than either the Sharpe or Treynor Indices. In addition, data envelopment analysis has also been used to evaluate the performance of mutual funds (Murthi, Choi, & Desai, 1997; Basso & Funari, 2001).

Several studies have dealt with the problem of portfolio selection by using various approximation schemes. Sharpe (1967) proposed a linear goal programming model for the portfolio selection of open-end mutual funds. Xia, Liu, and Lai (2000) proposed a new model for portfolio selection by considering the order of expected return which the investor is interested in. Xia, Wang, and Deng (2001), also applied a linear programming algorithm to study the optimal portfolio selection with transaction costs. Ballestero and Romero (1996) presented a compromise programming model for portfolio selection. Genetic algorithm (GA) and support vector machines (SVM) have also been adopted for solving portfolio optimization problems (Oh, Kim, & Min, 2005; Ince & Trafalis, 2006). While in a fuzzy environment, Tanaka and Guo (1999) presented possibility portfolio models to predict the state of future stock markets by integrating past data and expert's judgments. Parra, Terol, and Uria (2001) applied a fuzzy goal programming method to portfolio Spanish mutual funds selection. Carlsson, Fuller, and Majlender (2002) assumed that each investor can assign a welfare or utility score to computing investment portfolios based on the expected return and risk of those portfolios. Ammar and Khalifa (2003) introduced the formulation of fuzzy portfolio optimization problem based on the Markowitz's mean-variance model. In their model, the future return rates and variances are denoted by triangular fuzzy numbers. Referring to the Ammar and Khalifa (2003) model, we will develop our portfolio selection model and introduce its application to Taiwan's equity funds in the following sections.

3. Methodology

In this section, we develop a methodology to help investors in portfolio selection. Before that, we will examine the performance of funds based on four evaluation techniques. Then, applying those performance techniques, we proceed to cluster analysis to group a large amount of funds into several clusters based on their performance. The aim is to induce the characteristics of funds into groups to help investors in selecting which mutual funds to invest in.

3.1. Evaluation Indices

We choose four evaluation indices to perform the cluster analysis. The first one is the most direct and intuitive index, rate of return. The higher the rate of return, the better the performance of the fund is. The second index is the standard deviation of funds, which measures the volatility level of return rates; the volatility level also denotes the risk level. Usually, for higher rates of return the risk level will also become higher. Hence, we include standard deviation as the measure of risk level due to rates of return. The next index is turnover rate, denoting the changing in ratio of holding assets over a time period. A high turnover rate shows the aggressive operation of the funds, and hence causes an increase in transaction costs, while a low turnover rate shows stability in the performance of the funds. On average, a high turnover rate does not really benefit rate of return or other indices. The last index chosen is the Treynor index, which measures the excess return on every unit of systematic risk (β). Another common index, the Sharpe Index, is not used here, because past research has shown that the Sharpe and Treynor indices are just different approaches for the same end. As the amount of funds is large enough, investors only care about the systematic risks not the total risks, because total risks are assumed to be well-diversified. Therefore, the Sharpe index, which considers total risks, is not considered here, while the Treynor Index is chosen as instead. Each index is formulated as below.

(1) Rate of return
Rate of return, based on the concept of net asset value, is computed as

$$R_i = \frac{\mathrm{NAV}_{i1} - \mathrm{NAV}_{i0}}{\mathrm{NAV}_{i0}} \times 100\% \tag{2}$$

where i is the number of funds, R_i is the rate of return of portfolio i (in percentage), NAV_{i1} is the net asset value of funds at current evaluation period, NAV_{i0} is the net asset value of funds at previous evaluation period.

(2) Turnover rate

$$\text{Turnover rate} = \frac{\text{The amount of values in transactions (buy or sell)}}{\text{Average value of total assets}} \times 100\% \tag{3}$$

(3) Treynor index

$$T_i = \frac{\overline{R}_i - \overline{R}_f}{\beta_i} \tag{4}$$

where i is the number of funds; T_i is the Treynor's portfolio performance measure of portfolio i; β_i is the beta of portfolio i over the evaluation period; $\overline{R}_i$ is the average holding period returns on the portfolio i over the evaluation periods; $\overline{R}_f$ is the average risk-free return over the evaluation period.

(4) Standard deviation

$$\sigma_{i,\mathrm{MO}} = \sqrt{\frac{\sum_{j=1}^{m} (R_{ij} - \overline{R})^2}{m-1}} \tag{5}$$

$$\sigma_{i,\mathrm{Yr}} = \sigma_{i,\mathrm{MO}} \times \sqrt{12}$$

where $\sigma_{i,\mathrm{MO}}$ is the monthly standard deviation for portfolio i; $\sigma_{i,\mathrm{Yr}}$ is the yearly standard deviation for portfolio i; R_{ij} is the