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Particle swarm optimization approach to portfolio optimization

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ABSTRACT

The survey of the relevant literature showed that there have been many studies for portfolio optimization problem and that the number of studies which have investigated the optimum portfolio using heuristic techniques is quite high. But almost none of these studies deals with particle swarm optimization (PSO) approach. This study presents a heuristic approach to portfolio optimization problem using PSO technique. The test data set is the weekly prices from March 1992 to September 1997 from the following indices: Hang Seng in Hong Kong, DAX 100 in Germany, FTSE 100 in UK, S&P 100 in USA and Nikkei in Japan. This study uses the cardinality constrained mean-variance model. Thus, the portfolio optimization model is a mixed quadratic and integer programming problem for which efficient algorithms do not exist. The results of this study are compared with those of the genetic algorithms, simulated annealing and tabu search approaches. The purpose of this paper is to apply PSO technique to the portfolio optimization problem. The results show that particle swarm optimization approach is successful in portfolio optimization.

1. Introduction

The particle swarm optimization (PSO) approach is a heuristic technique introduced comparatively recently by Kennedy and Eberhart [1]. There are very few studies on PSO in the literature, and almost none of them deals with portfolio optimization (PO). This study presents a new approach to PO using PSO.

PO consists of the portfolio selection problem in which we want to find the optimum way of investing a particular amount of money in a given set of securities or assets [5]. Although the task of yielding minimum risk and maximum return looks simple, there is more than one way of establishing an optimum portfolio. Markowitz [2,6] formulated the fundamental theorem of a mean–variance portfolio framework, which explains the trade-off between mean and variance, representing expected returns and risk of a portfolio, respectively. An advanced model was introduced by Konno and Yamazaki [3] in which a mean-absolute deviation (MAD) model and absolute deviation are utilized as a measure of risk. However, it was insensitive to some extremes, which could be the source of serious error, contrary to the suggestion that the MAD model is suitable under all circumstances [7]. As Mansini and Sprenza stated [8], most of the portfolio selection models assume a perfect fractionability of the investments; however, securities are negotiated as multiples of a minimum transaction lot in the real world, and they suggested a mixed integer programming model with minimum lot constraint for portfolio selection.

Some researchers have investigated the multi-period PO case, in which investors invest continuously rather than at intervals or only once. Celikyurt and Ozekici [15] accomplished this, assuming that there are some economic, social, political and other factors affecting the asset returns. They formed their stochastic market with respect to these factors, and they used a Markov chain approach in their study.

This study basically employs the Markowitz mean–variance model. However, the standard model does not contain any cardinality or bounding constraints, which restrict the number of assets and, the upper and the lower bounds of proportion

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of each asset in the portfolio, respectively. Chang et al. [9] and Fernandez and Gomez [5] used a modified Markowitz model called a "cardinality constrained mean–variance (CCMV) model". Here, the CCMV model is used and is solved by a PSO approach.

There are some reports of solving the PO problem using heuristic methods. These methods consist of genetic algorithms (GA) [9,6,10], tabu search (TS) [9], simulated annealing (SA) [9,11,12], neural networks [5] and others [13,8,14]. The results of this study are compared with those of the GA, SA and TS approaches [9]. The test data set is the weekly prices from March 1992 to September 1997 from the following indices: Hang Seng in Hong Kong, DAX 100 in Germany, FTSE 100 in UK, S&P 100 in USA and Nikkei in Japan. The number of different assets for each of the test problems is 31, 85, 89, 98, and 225, respectively.

2. CCMV model for PO

This study uses the CCMV model [9] and [5], which is derived from the well-known Markowitz standard model, which is:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{1}$$

$$\text{subject to } \sum_{i=1}^{N} x_i \mu_i = R^*, \tag{2}$$

$$\sum_{i=1}^{N} x_i = 1, \tag{3}$$

$$0 \le x_i \le 1. \quad i = 1, \dots, N \tag{4}$$

where N is the number of different assets, σ_{ij} is the covariance between returns of assets i and j, x_i is the proportion of asset i in the portfolio, μ_i is the mean return of asset i and R^* is the desired mean return of the portfolio.

In order to observe the different objective function values for varying R^* values, standard practice introduces a risk aversion parameter $\lambda \in [0, 1]$. With this new parameter, the model can be described as:

$$\min \lambda \left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} x_i \mu_i\right] \tag{5}$$

$$\text{subject to } \sum_{i=1}^{N} x_i = 1, \tag{6}$$

$$0 \le x_i \le 1 \quad i = 1, \dots, N. \tag{7}$$

When λ is zero, the model maximizes the mean return of the portfolio, regardless of the variance (risk). In contrast, when λ equals unity, the model minimizes the risk of the portfolio regardless of the mean return. So, we can say that the sensitivity of the investor to the risk increases as λ approaches unity, while it decreases as λ approaches zero.

Each case with different λ value would have a different objective function value, which is composed of mean return and variance. Tracing the mean return and variance intersections, we draw a continuous curve that is called an efficient frontier in the Markowitz theory [2]. Since every point on an efficient frontier curve indicates an optimum, the PO problem is a multi-objective optimization problem. So, a definition must be adopted for the concept of optimal solution. This study used the Pareto optimality definition, which questions whether a feasible solution of the PO problem will be an optimal solution (or non-dominated solution) if there is no other feasible solution improving one objective without making the other worse [5]. For the problem defined in Eqs. (5)–(7), the efficient frontier is a curve that gives the best trade-off between mean return and risk. Fig. 1 shows such a curve corresponding to the smallest benchmark problem (Hang Seng) described in Section 4. This efficient frontier has been computed taking 2000 different λ values; that is, there have been 2000 distinct objective function values for the resulting solutions. Thus, each of the solutions corresponds to a point in the efficient frontier. This curve was called a standard efficient frontier by Fernandez and Gomez [5].

Some additional variables have to be included in the standard model in order to describe the CCMV model. As mentioned above, there are two constraints in the CCMV model in addition to those of the original model. The first one is to restrict K, the number of assets in the portfolio. If the decision variable $z_i \in \{0, 1\}$ is 1, asset i will be included in the portfolio, otherwise it will not be. The second constraint is that an included asset's proportion is within the lower and the upper bounds, ε_i and δ_i, respectively. Thus, the CCMV model is:

$$\min \lambda \left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} x_i \mu_i\right] \tag{8}$$

$$\text{subject to } \sum_{i=1}^{N} x_i = 1, \tag{9}$$