Economic Modelling 28 (2011) 1482–1488

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Central banks and their banknote series: The efficiency–cost trade-off

Y. Bouhdaoui [a,1], D. Bounie [a,*], L. Van Hove [b,2]

[a] Telecom ParisTech, Economics and Social Sciences (ESS), 46 rue Barrault, 75634, Paris Cedex 13, France
[b] ECON–2C147, Vrije Universiteit Brussel (Free University of Brussels), Pleinlaan 2, B-1050 Brussels, Belgium

ARTICLE INFO

Article history:
Accepted 5 February 2011

JEL classification:
E4
E42
E47

Keywords:
Currency
Central banks
Efficient payments
Production costs
Simulation

ABSTRACT

One of the most important results of theoretical research on currency systems is that spacing denominations apart by a factor of two is better than a factor of three as this lowers the average number of notes and coins exchanged in transactions. These theoretical studies also claim that an efficient denominational mix has the additional benefit of keeping down the production costs incurred by the central bank. This paper challenges this claim and demonstrates that more efficient currency systems can also be more costly. Central banks therefore face an efficiency–cost trade-off and have to weigh the benefits for transactors against those for the central bank itself.

1. Introduction

In the theoretical research on optimal denominations for coins and banknotes, efficiency has always been the predominant concern. This is reflected in the popularity of the “principle of least effort”, which holds that a denominational structure should make it possible for transactors to economize on the number of tokens exchanged. For the case of exact payment, Caianiello et al. (1982) demonstrated early on that modular currency systems – systems in which each denomination is X times the one below it (with X an integer) – are the most efficient, and that the number of tokens exchanged is a growing function of the spacing factor. For the case where overpayment and the return of change is allowed, Van Hove and Heyndels (1996) and Van Hove (2001) eventually, after many a controversy, showed that the optimal spacing factor is *not* three – as claimed by Sumner (1993) and Telser (1995) – but rather two, as in the case of exact payment.

These theoretical studies supposedly also have important practical implications beyond efficiency. In particular, a reduction in the number of tokens exchanged is assumed not only to be more convenient for transactors, but also to keep down the number of coins and banknotes in circulation, and thus the handling and production costs incurred by the central bank.[3] In short, the most efficient denominational mix would at the same time also be the most cost efficient, at least for the central bank. This matters because the currency expenses of central banks are quite high. In the U.S, for example, the cost for the Federal Reserve System of new currency alone is budgeted at $703 million for 2010, which is equivalent to 16% of the total budget.[4]

The objective of this paper is to show that reducing the spacing factor can actually *increase* the production costs incurred by the central bank. To that end, we build on Cramer's model of efficient payments (Cramer, 1983) by incorporating the production costs of the tokens. The resulting model makes explicit how an increase in the “density” of a currency system – that is, an increase in the number of denominations over a given interval – on the one hand improves its efficiency and thus lowers the variable production costs, but on the other hand increases the fixed production costs. Using simulations in which we compare powers-of-two and powers-of-three currency systems, we demonstrate that, under certain conditions, the lower average frequency of use that comes with the powers-of-two system is not sufficient to offset the second effect. In other words, while the powers-of-two system is clearly more efficient, it can be more costly

* Corresponding author. Tel.: +33 1 45 81 73 32.
E-mail addresses: yassine.bouhdaoui@telecom-paristech.fr (Y. Bouhdaoui), david.bounie@telecom-paristech.fr (D. Bounie), Leo.Van.Hove@vub.ac.be (L. Van Hove).
[1] Tel.: +33 1 45 81 82 95.
[2] Tel.: +32 2 629 21 25.

[3] Many economists support this view; see Boeschoten and Fase (1989), Eriksson and Kokkola (1993), Abrams (1995), Pedersen and Wagener (1996), Van Hove and Heyndels (1996) and Van Hove (2001).

[4] Source: Board of Governors of the Federal Reserve System, Annual Report: Budget Review, May 2010, p. 9 <http://www.federalreserve.gov/boarddocs/rptcongress/budgetrev/br10.pdf>.

0264-9993/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.econmod.2011.02.029

for central banks than a powers-of-three system, and this even with identical cost structures. Obviously, pure powers-of-two or powers-of-three systems – the latter having denominations of 1, 3, 9, 27, 81, etc. – are not really viable in practice.[5] We have therefore also included two real-life currency systems in our simulations, namely the series used by the European Central Bank (ECB) and the Federal Reserve. Again we find that the central bank faces an efficiency–cost trade-off and thus has to weigh the benefits for transactors against those for the bank itself.

The paper contributes to the literature on optimal denominations for coins and banknotes on two crucial points. For one, it is the first to fully consider all production costs. Indeed, while Bouhdaoui and Bounie (2010) also show that the principle of least effort fails to minimize the production costs of the central bank and that it should thus be reconciled with cost considerations, they use a basic cost function of the central bank that ignores the fixed costs of production. As a result, they do not take into account economies of scale, which are critical in understanding the design of a denominational mix. Second, the present paper looks into the issue of what constitutes a (theoretically) optimal denominational mix. Bouhdaoui and Bounie (2010), for their part, only compare the cost of efficient and inefficient payments for a single real-life currency system, namely the U.S. series as it exists today.

The remainder of the paper is structured as follows. In Section 2, we present an original framework that models the currency production costs of the central bank and in particular indicates how these are affected by the frequency of use of the tokens in circulation. In Section 3, we use this framework to study – still on a purely theoretical level – the conditions under which one currency system can be more costly than another. In Section 4, we then illustrate our analysis by performing simulations for selected currency systems. Section 5 discusses the implications of our results.

2. General framework

In this section, we introduce our general framework — in two steps. We first present a currency production cost function and discuss the different types of costs involved. Subsequently, we model how these production costs are affected by parameters such as the number of cash transactions in the economy, the quality standards of the central bank, and especially the efficiency of the currency system.

2.1. The production costs of the central bank

In a given economy, let us consider a central bank that has a currency system V composed of J denominations of face values $v(j)$, with $j \in [1, J]$. Let us assume that the central bank mints its own coins and prints its own banknotes. If we denote by $N_r(j)$ the volume of new tokens of a specific denomination that are issued by central bank in a period of, say, one year, and if we assume uniform amortization, then the total annual production cost C incurred by the central bank for these tokens can be written as[6]:

$$C = C_0^F + \sum_{j=1}^{j=J} \left(C_p^F(j) + c_p^V(j) \cdot N_r(j) \right). \qquad (1)$$

In this equation, C_0^F stands for the “overall” fixed costs, $C_p^F(j)$ for the fixed costs on the level of an individual denomination, and $c_p^V(j)$ for the unit variable cost of a denomination. Note that the distinction between fixed and variable costs – and especially the presence of $C_p^F(j)$ – will prove crucial for our analysis of the cost efficiency of currency systems in Section 3. Below we discuss and illustrate the different types of costs in more detail. For reasons of brevity, we do this for the case of banknotes, and not for coins. Illustrations are mainly based on data from the U.S. Bureau of Engraving and Printing (BEP) – the government agency responsible for the development and production of U.S. currency notes – and the Federal Reserve, the central bank that is most transparent in this respect.

As is clear from Eq. (1), overall or “high-level” fixed costs are fixed costs that the central bank incurs regardless of the number of denominations in its currency system. Infrastructure and indirect personnel costs come to mind. In its annual “New Currency Budget” (NCB), the Federal Reserve does not split up its printing costs into fixed vs. variable costs, but it does provide a split-up into “capital investment”, “public education”, “production support”, and “currency production”. Of these four cost components, the first two are clearly fixed. The percentage contribution of these and other factors varies from one year to another – depending for example, on the number of notes ordered by the Fed – but in 2009 capital investment represented approximately 6% of total currency printing costs and public education 2%.[7] The item “production support” (46% in 2009) is of a mixed nature: it comprises such fixed outlays as expenses for IT security and business continuity, but costs for “steam, natural gas, water and electricity” are influenced by production volume. Dan Peterson of the BEP estimates that the item is “probably 70% fixed and 30% variable”.[8] Finally, “currency production” – 46% of the total in 2009 – is mainly about variable costs (paper, ink, the cost of note packaging, etc.), but also includes depreciation for the printing/processing equipment.[9] Separately from the NCB, the BEP kindly provided us with detailed data on the manufacturing cost of U.S. banknotes, and this per denomination and per production facility (the BEP has two separate production facilities).[10] We have used the BEP data for September 2009 to calculate a weighted average of the share of depreciation in the manufacturing cost and arrived at 4.8%. In total, as a rough estimate, fixed costs would thus have accounted for 42.4% in 2009 (=6% + 2% + 70%*46% + 4.8%*46%).[11]

As mentioned, Eq. (1) not only distinguishes fixed from variable costs, but also makes a distinction between overall fixed costs and fixed costs per denomination. Where the latter are concerned, there is for instance, an “origination cost” attached to the creation of a new denomination. This relates to “the high-quality image preparation and the transformation of designs into production tools, such as printing plates and production forms for the different manufacturing steps” (European Central Bank, 2007, p. 38).[12] Expenses on research and development – for example concerning security features – may constitute another type of fixed cost per denomination. In the U.S., the BEP for example “incurred significant fixed research and development costs in 2009 associated with testing the new-design $100 notes”.[13] However, sometimes such R&D costs are not incurred for a single denomination but rather intended for and therefore amortized over

[5] Tschoegl (1997: 546) notes that today no currency follows the powers-of-three principle, and that even in the past only few examples of the use of the principle can be found. Wynne (1997: 222) finds that only 5 countries have denominations that are either powers or integer multiples of three.

[6] For simplicity, we ignore the fact that in certain years the central bank might produce more tokens of a certain denomination than it really needs (in order to build a buffer stock), while in other years it will draw on this stock.

[7] Sources: 2009 NCB.

[8] Source: Peterson, D., private e-mail, January 24, 2011 (on file with the authors).

[9] Source: Peterson, D., private e-mail, January 24, 2011 (on file with the authors).

[10] Source: BEP. (2009). Financial Report for Monthly Production and Cost Meeting — Month of September 2009.

[11] Note that this should be seen as a lower limit. Indeed, as Dan Peterson pointed out to us, it could be argued “that the direct labor [which is included in “currency production”] in the short term is fixed. The BEP would be reluctant to furlough its skilled workforce” (Source: Peterson, D., private e-mail, January 24, 2011 (on file with the authors)). It proved impossible to determine the relative importance of direct labor with the data that we have.

[12] The issuance of a new denomination may also require new public education materials, so that it could be argued that this item too is – in part – a per-denomination fixed cost.

[13] Source: 2010 NCB.