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An equity-efficiency trade-off in a geometric approach to committee selection

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ABSTRACT

The trade-off between equity and efficiency is analyzed in a geometric framework for the problem of committee selection, which has recently attracted interest in the social choice literature. It is shown that this trade-off can be maximal in the precise sense of the antipodality of the outcomes corresponding to the rules implementing the two normative principles. Following an approach in location theory, the minimization of the convex combination of the two criteria is presented as a compromise solution.

1. Introduction

The equity-efficiency trade-off is one of the major puzzles in the ethical evaluation of economic outcomes and the diverse mechanisms—such as markets or voting rules—supporting those outcomes. This trade-off is also related to the conflict between the two most prominent welfare criteria in welfare economics and social choice theory, Rawlsian egalitarianism and utilitarianism à la Harsanyi.[1]

Following the seminal work of Saari (1995), a geometric approach has been introduced into social choice theory. In particular such an approach provides a framework for the comparison of voting rules with the help of the distance between their respective outcomes (for such comparisons of voting rules see Eckert and Klamler (2008), Klamler (2004), and Ratliff (2002)). A geometric approach is particularly natural if the preferences can be assumed to be induced by the topology of the space of alternatives. In this case, complete preferences can be derived from exclusive information about a voter's most preferred ("ideal") alternative. Typically this is done in spatial voting, where Euclidean preferences are obtained by identifying alternatives with a point in the d-dimensional Euclidean space $\mathfrak{R}^d$ and ranking them according to their distance to an individual's ideal point. Similar issues being analyzed in location theory, where costs are derived from distances (see Nickel and Puerto (2005) and Daskin (1995)), we will adapt an approach from location theory to distance-based voting.

As an application, we use the problem of committee selection which has recently been analyzed in a distance-based framework. In Brams et al. (2007) a committee selection rule is a function that assigns to each profile of voters' ideal committees a committee that minimizes a distance-based objective function. In particular they have introduced a minimax procedure and

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[1] For two excellent introductions into this area from a social choice and a moral philosophy point of view, respectively, see Moulin (2003) and Hausman and McPherson (2006).

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compared it with the application of majority voting to the election of committee members. Their finding, that these two rules can lead to antipodal results is all the more disturbing as these rules can be considered to incorporate the principles of equity and of efficiency respectively. We extend their approach and suggest the use of a classical solution from location theory to deal with this problem.

2. Equity-efficiency trade-off: formal framework and antipodality result

For a finite set of candidates, J, a committee can be represented by a vector of binary valuations $x=(x^1, x^2, \ldots, x^{|J|}) \in X \subseteq \{0, 1\}^{|J|}$, where $x^j=1$ means that candidate j belongs to the committee and X denotes the set of all committees under consideration.

Vectors of binary valuations are very flexible instruments for the representation of various inputs in social choice problems, e.g. ordered pairs of alternatives in classical Arrovian preference aggregation or sets of propositions in the recent field of judgment aggregation (see Dokow and Holzman (2010), Rubinstein and Fishburn (1986)). Hence the results of this paper carry over to any social choice problem which can be formulated within this very general framework.

Geometrically, X is a subset of the vertices of a $|J|$-dimensional hypercube. Possible restrictions on the set X, which are ignored here, can guarantee non-emptiness and fixed sizes as well as specific compositions of committees, e.g. desirable combinations of particular members. On the other hand, the absence of restrictions on the set of binary valuations establishes a certain analogy to approval voting, where the voters can approve of any number of candidates.[2]

We assume that the individual voters have metric preferences over committees, i.e. preferences that can be derived from an individual's ideal committee and a distance function $d{:}X \times X \rightarrow \mathfrak{R}_+$ over the binary valuations representing the alternative committees. An individual with ideal committee x^*, will then consider committee y at least as good as committee z if and only if $d(x^*, y) \leq d(x^*, z)$. While Euclidean preferences are the most important class of metric preferences (see Bogomolnaia and Laslier (2007)), the most widely used distance function for binary valuations is the Hamming distance. Here, for any committees x, $y \in X$, $d(x, y)$ is the number of components (candidates) in which the committees x and y differ. For example, the distance between the committees $x=(1, 0, 0, 0)$ and $y=(0, 1, 0, 1)$ is $d(x, y)=3$. (Observe that the set X can be embedded into the $|J|$-dimensional Euclidean space which makes the Hamming distance equivalent to the Manhattan distance or l_1-norm.)

As a typical limitation of the use of metric preferences, it must be noted that the Hamming distance is not responsive to complementarities between the candidates, for example a concern for gender balance. E.g., if $J=\{$Ann, Betty, Chris, Dan$\}$ there is good reason to assume that a voter with ideal committee $x^*=(1, 0, 1, 0)$ who is concerned with gender balance will prefer committee $y=(0, 1, 0, 1)$ to $z=(1, 1, 0, 0)$ although the Hamming distance between y and x^* is twice as large as the one between z and x^*.

A highly desirable property for voting rules is the condition of anonymity which guarantees that all voters have equal weight. If this property is imposed on committee selection, a profile of voters' characteristics can be represented by a vector $\boldsymbol{p}=(p_1, p_2, \ldots, p_{|X|}) \in [0, 1]^{|X|}$ which associates with every binary valuation $x_k \in X$ the fraction p_k of individuals for which x_k is the ideal committee, where $\sum_{i=1}^{|X|} p_i=1$.

A committee selection rule f is a mapping that assigns to every profile in vector representation $\boldsymbol{p}=(p_1, p_2, \ldots, p_{|X|})$ a committee $x=f(\boldsymbol{p})$.[3]

Various social choice rules are distance-based (such as the Kemeny rule) or can be characterized or rationalized with the help of distance functions (see Nurmi (2004)). Such distance-based choice rules are particularly natural if metric preferences are already assumed.

In the context of committee selection, two distance-based choice rules were introduced by Brams et al. (2007): the minimization of the average distance to the individuals' ideal committees (minisum) on the one hand and the minimization of the maximal distance between the selected and the ideal committee of some individual on the other hand (minimax).

In location theory these rules are known as the median objective function and the center objective function for locating a facility such that customers are "best" served (see Daskin (1995)).

Normatively, these rules can be seen to implement the principles of efficiency and equity, respectively, as the sum of (dis) utilities is a criterion of efficiency in a utilitarian perspective while the position of the least advantaged individual is a criterion of equity in the spirit of Rawls (1999).

Definition 1. For any profile $\boldsymbol{p}=(p_1, p_2, \ldots, p_{|X|})$, the minisum outcome $^S x$ is the committee that, among all $y \in X$, minimizes $\sum_{i=1}^{|X|} p_i d(x_i, y)$.

It is easily seen that the minisum rule is a metric rationalization of majority voting on candidates: for any profile $\boldsymbol{p}=(p_1, p_2, \ldots, p_{|X|})$ the outcome of committee selection by majority voting on candidates, $^S x=MS(\boldsymbol{p})$, is defined as follows:

Definition 2. For any candidate $j \in J$, $^S x^j=1$ if and only if $\sum_{i=1}^{|X|} p_i x_i^j > 0.5$.

Thus committee selection via majority voting on candidates can be implemented by the minisum rule and hence inherits its normative appeal.

2 On approval voting see Brams and Fishburn (1983), on the welfare effects of this voting rule see Lehtinen (2008).

3 Possible restrictions on the codomain of the committee selection rule, such as fixed numbers of committee members or the exclusion of certain combinations of members are again ignored here. On the other hand, our framework makes the size of the committee endogeneous.