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The Geometric Portfolio Optimization with Semivariance in Financial Engineering

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Abstract

In this paper we consider a portfolio optimization problem on maximizing the geometric mean return subject to the lower semivariance as a risk measure in the financial engineering. Its optimal condition and the solving method via the Monte Carlo simulation are given, and a numerical experiment is presented in order to show that the method is efficient.

Keywords : Financial engineering; Portfolio optimization; Monte Carlo simulation; Numerical experiment

1. Introduction

The geometric mean investment strategy, introduced into the finance and economics literature by Henry Latane [1] in 1959, has recently received some attentions in scholarly circles [2-5]. Ye and Li [6] considered the geometric mean return on portfolio investments with the variance of returns as a risk measure. The variance, however, is a questionable measure of risk for at least two reasons: First, it is an appropriate measure of risk only when the underlying distribution of returns is symmetric. And second, it can be applied straight forwardly as a risk measure only when the underlying distribution of returns is normal. However, both the symmetry and the normality of stock returns are seriously questioned by the empirical evidence on the subject. The lower semivariance of returns, on the hand, is a more plausible measure of risk [7-9] for several reasons: first, investors obviously do not dislike upside volatility; they only disliked own side volatility. Second, the lower semivariance is more useful than the variance when the underlying distribution of returns is asymmetric and just as useful when the underlying distribution is symmetric; in other words, the lower semivariance is at least as useful a measure of risk as the variance. And third, the lower semivariance measures the information provided by two statistics, variance and skewness, thus making it possible to use a one-factor model to estimate required returns. Thus, we will consider the lower semivariance as a risk measure to maximize the geometric mean return on portfolio investments. The paper is organized as follows. Section 2 develops our portfolio optimization model and its optimal condition. In Section 3, a Monte Carlo method is proposed to solve the model and a numerical example is given to show the effectiveness of the model. Conclusion is given in Sections 4.

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2. Model and optimality condition

2.1. Model

Consider a single-period financial investment problem. Suppose that a investor (individual or company) invests one unit capital in finite assets in a capital market. If x_i denotes the fraction of his wealth which the investor invests in asset i and $\eta_i > 0$ denotes the periodic payout, including return of principal, provided by asset i , $i = 1, 2, \cdots, n$. Let $\boldsymbol{x} = (x_1, \cdots, x_n)^{\mathrm{T}}$ be a portfolio vector and $\boldsymbol{\eta} = (\eta_1, \cdots, \eta_n)^{\mathrm{T}}$ be the random vector defined on a probability space $(\Omega, \boldsymbol{F}, P)$ and has a continuous distribution function $\boldsymbol{F}(\cdot)$ with its probability density function $p(\boldsymbol{\eta})$ .It is easy to derive the formula for the total return

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta} = \sum_{i=1}^{n} x_i \eta_i \tag{1}$$

Furthermore, the geometric mean return on portfolio investments is denoted by

$$\psi(\boldsymbol{x}) = E[\log(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta})] \tag{2}$$

where $E[\cdot]$ is a expectation operator.

It is possible that the value of the total return $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta}$ fluctuates since it is uncertain at the end of the period, and the negative deviation from the expected return value is a risk for the averse risk investor. So, we assume that the investor uses the lower semivariance as the risk measure, whose definition is given as follows

$$SV(\boldsymbol{x}) = E[(\min\{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta} - E(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta}), 0\})^2] \tag{3}$$

Clearly, the set of possible asset allocations can be defined as follows

$$D = \{\boldsymbol{x} / \sum_{\mathrm{i}=1}^{\mathrm{n}} x_i = 1, x_i \geq 0, i = 1, \cdots, n\} \tag{4}$$

Thus, the portfolio optimization problem on maximizing the geometric mean return under the lower semivariance constraint is formulated as follows

$$\begin{aligned} &\max_{\boldsymbol{x}\in D} \ \psi(\boldsymbol{x}) = E[\log(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta})] \\ &\text{s.t. } SV(\boldsymbol{x}) \leq \gamma \end{aligned} \tag{5}$$

where γ is a constant presenting a risk constrain level.

Let $\varphi(\boldsymbol{x}) = -\psi(\boldsymbol{x})$. Then Eq. (5) is equivalent to the following problem

$$\begin{aligned} &\min_{\boldsymbol{x}\in D} \ \varphi(\boldsymbol{x}) = -E[\log(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\eta})] \\ &\text{s.t. } SV(\boldsymbol{x}) \leq \gamma \end{aligned} \tag{6}$$

2.2. Optimality condition

The following lemma is important to prove the optimality condition, and see [10].