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Portfolio Optimization with Cardinality Constraints Based on Hybrid Differential Evolution

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Abstract

A portfolio optimal model with cardinality constraints is researched, in which the minimum of Value-at-Risk is taken as the objective function. We give a hybrid differential evolution algorithm to solve the model and make the case study with sixteen alternative stocks from Shanghai and Shenzhen stock market. The numerical results show that the given model is reasonable and the given algorithm is effective.

Keywords:Portfolio optimization, Value-at-Risk(VaR), Cardinality constraints, hybrid differential evolution

1. Introduction

Since 1970s, financial risk has emerged increasingly due to the economic globalization, the rapid development of the modern information technology and financial system innovation. How to control financial risks effectively becomes the focus to financial regulators and investors. Therefore, how to measure risk has been a very important research subject.

In 1952, Markowitz [1] proposed the mean-variance model in which the variance is risk measure. Markowitz [2] and Mao [3] discussed the mean semi-variance portfolio model in which the semi-variance is

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risk measure. In 1991, Konna and Yamazaki [4] adopted mean semi-absolute deviation as the risk measure and proposed a linear programming model of portfolio, which is called as the mean - absolute deviation model. This model can be solved effectively by linear programming method. But the study only measured the deviation level of future returns and expected returns, and didn't make comprehensive accurate quantitative calculation to risk of loss. Consequently, the Group of Thirty proposed a new risk measure tool named VaR [5] in 1993. VaR reflects a potential biggest loss of an asset or portfolio in a certain period and at the confidence level. But VaR does not satisfy subadditivity, so it is not a coherent risk measure. Uryasev and Rockafellar [6] proposed Conditional Value-at-Risk (CVaR) portfolio model and obtained approximate conclusion with VaR.

In recent years, there has been much research on portfolio model based on VaR and CVaR which consider the actual market friction, such as transaction costs. But, research on portfolio with cardinality constraints is still not enough, for it is a mixed integer programming problem and difficult to solve by traditional methods. Paper [7] proposed heuristic algorithm for the portfolio optimization problem. Then, Paper [8] used local hybrid search algorithm to solve portfolio optimization problems with cardinality constraints.Yi Wang and Zhiping Chen [9] studied portfolio problem with cardinality constraints under multivariate distribution.

We apply differential evolution algorithm to solve the problem in order to obtain reasonable results which ensured the feasibility of our solution. The numerical results show that the model is reasonable and the given hybrid algorithm is effective. That is to say, this paper provides an effective method for the portfolio optimization model with cardinality constraints.

2. M-VaR model with cardinality constraints

2.1. Value-at-Risk Description

Suppose $f(x,y)$ is a loss function of decision vector, x is the vector of portfolio weights, $x=[x_1,x_2,\cdots,x_n]$, where n is the number of available assets, x_i ($i=1,2,\cdots,n$) is the portfolio weight of the i th asset, and $0 \le x_i \le 1, \sum_{i=1}^{n} x_i = 1$. y is the vector of market factor, such as market price, ratio of return. For any x, the loss $f(x,y)$ caused by Vector y is a random variable and follows a distribution in R .Suppose probability density function of y is $p(y)$, then the probability of $f(x,y)$ doesn't exceed Δp is:

$$\psi(x,\Delta p)=\int_{f(x,y)<\Delta p} p(y)dy \tag{1}$$

As a function of Δp, when x is fixed, $\psi(x,\Delta p)$ is a cumulative distribution function of x .It is right continuous and non-decreasing with respect to Δp. Then VaR can be denoted by VaR_α, α is the confidence level. In our setting they are given by:

$$VaR_\alpha = \min\{\Delta p \in R : \psi(x,\Delta p) \ge \alpha\}$$