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Combining equilibrium, resampling, and analyst's views in portfolio optimization

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ABSTRACT

This paper proposes the use of a portfolio optimization methodology which combines features of equilibrium models and investor's views as in [Black and Litterman \(1992\)](#), and also deals with estimation risk as in [Michaud \(1998\)](#). In this way, our combined methodology is able to meet the needs of practitioners for stable and diversified portfolio allocations, while it is theoretically grounded on an equilibrium framework. We empirically test the methodology using a comprehensive sample of developed countries fixed income and equity indices, as well as sub-samples stratified by geographical region, time period, asset class and risk level. In general, our proposed combined methodology generates very competitive portfolios when compared to other methodologies, considering three evaluation dimensions: financial efficiency, diversification, and allocation stability. By generating financially efficient, stable, and diversified portfolio allocations, our methodology is suitable for long-term investors such as Central Banks and Sovereign Wealth Funds.

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1. Introduction

Portfolio optimization methodologies play a central role in strategic asset allocation (SAA) where it is desirable to have portfolios that are efficient, diversified, and stable. Since the development of the traditional mean–variance approach of [Markowitz \(1952\)](#), many improvements have been made to overcome problems, such as lack of diversification and strong sensitivity of optimal portfolio weights to expected returns.

The [Black and Litterman \(1992\)](#) model (hereafter BL) is among the most used approaches. The main idea of this model is that expected returns are the result of two important sources of information: market information in the form of equilibrium returns (implicit returns that clear out the outstanding market allocation), and analysts' views which tilt the market portfolio to another diversified portfolio compatible with investor beliefs. In this fashion, portfolio managers get an intuitive but formal model to generate optimal allocation.

However, while the BL model offers a very useful and intuitive approach to deal with asset allocation, the inputs considered for

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the calculation of equilibrium returns are subject to estimation error. [Michaud \(1998\)](#) proposed the use of resampling to deal with estimation error, which is an important source of lack of diversification in mean–variance portfolio. This technique considers that data come from a stochastic process instead of being a deterministic input as in [Markowitz \(1952\)](#).

This paper proposes the use of a portfolio optimization methodology which combines features of both BL and resampling methodologies. This methodology allows a novel combination of equilibrium and investor's views as in BL, and at same time, deals with estimation risk as in [Michaud \(1998\)](#). Thus, it generates robust and diversified optimal allocations which are desirable properties for long-term investors such as Central Banks and Sovereign Wealth Funds. We empirically test this methodology using a sample of fixed income and equity indices, achieving very supportive results. We find strong evidence supporting the use of resampling techniques to improve standard models like BL and Markowitz. In general, our proposed combined methodologies, both with and without views, generated very competitive portfolios compared to the other methodologies, considering the three evaluation dimensions: financial efficiency, diversification, and allocation stability.

The remainder of this paper is as follows. Next section offers a brief literature review over asset allocation methodologies. The third section describes the Black-Litterman-Resampling combined methodology. The fourth section describes the empirical study,

including data, implementation and initial results. Section 6 presents the robustness checks and Section 7 concludes the paper.

2. Literature review

The seminal work of Markowitz (1952) provided the first model for asset allocation, arguing that once expected returns and their joint variance were defined, a set of efficient portfolios could be generated and investors would choose the allocation according to their needs. Basically the approach can be summarized as follows:

$$\begin{aligned} & \text{Min}(1/2)\mathbf{a}^T\mathbf{V}\mathbf{a} & (1) \\ & \text{subject to} \\ & E[\mathbf{R}_a] = \mathbf{a}^T\mathbf{X}, \end{aligned}$$

where \mathbf{X} is the vector of expected excess asset returns, \mathbf{a} is the vector of allocations, and \mathbf{V} is the variance–covariance matrix of asset returns. Despite its mathematical simplicity, this model typically generates concentrated allocations which heavily depend on expected returns estimation. Resampling techniques (Michaud, 1998) were developed as a way to deal with estimation error. Markowitz recognized that resampling methods could be used to obtain better estimates for the inputs of the mean–variance optimization (Markowitz and Usmen, 2003).

Jorion (1991) used the Bayesian approach to overcome the weakness of expected returns estimated solely by sample information. He proposed an estimator obtained by “shrinking” the mean values toward a common value, chosen to be the expected return for the minimum variance portfolio. Kempf et al. (2002) applied Bayesian methods and considered estimation risk as a second source of risk, determined by the heterogeneity of the market, which is represented by the standard deviation of the expected returns across risky assets. Both methods proved to generate better out-of-sample estimates for expected returns (as opposed to in-sample estimates), and also produced more diversified portfolios.

Black and Litterman (1992) built a bridge between statistical methods and expert judgment by recognizing that capital asset pricing model (CAPM) offers an appropriate starting point for expected excess returns. Thus, combing CAPM with investors’ views would produce intuitive and diversified allocations. For that, BL assume that equilibrium returns (CAPM returns that clear out the market) are well described by the following relationship:

$$\mathbf{X} \sim N(\mathbf{\Pi}, \tau\mathbf{\Sigma}), \tag{2}$$

where \mathbf{X} is the observed returns vector which is just a realization of the multivariate normal process with mean $\mathbf{\Pi}$ (equilibrium returns), covariance matrix $\mathbf{\Sigma}$ and an scale parameter τ which measures the degree of confidence the investor has on equilibrium estimates (the closer the parameter is to zero, the higher is the confidence in equilibrium estimates).

In addition to this, BL postulate that returns have another important source of information, coming from investor’s views:

$$\mathbf{X} \sim N(\mathbf{Q}, \mathbf{\Omega}), \tag{3}$$

where \mathbf{Q} denotes the vector of expected return views (this could be absolute or relative) and $\mathbf{\Omega}$ is the uncertainty in those views. Since $\mathbf{\Omega}$ is not an easy-to-obtain parameter, we employ the Idzorek (2004) approach which measures the uncertainty through a degree of confidence and implicitly calculates $\mathbf{\Omega}$. With both sources of information, the combined process is also a multivariate normal as follows:

$$\mathbf{X} \sim N([\mathbf{(\tau\Sigma)^{-1} + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}]^{-1}[\mathbf{(\tau\Sigma)^{-1}\mathbf{\Pi} + \mathbf{P}^T\mathbf{\Omega}\mathbf{Q}], [(\mathbf{\tau\Sigma})^{-1} + \mathbf{P}^T\mathbf{\Omega}^{-1}\mathbf{P}]^{-1}], \tag{4}$$

where \mathbf{P} denotes the portfolio view matrix whose dimension is a function of the number of views (rows) and the number of assets

(columns). Needless to say is that, since market capitalization offers a well-diversified portfolio, the optimal allocation (in general) will have this property with tilts reflecting investors’ views introduced in the model.

Finally, Michaud (1998) adapted the resampling statistic technique to mean–variance optimization recognizing that return history is just a realization of the stochastic process behind it. Also, if stationarity holds and in a large sample environment, the point estimates could statistically resemble the true distribution parameters. Suppose that we have a vector of expected excess return \mathbf{X}_0 and a variance–covariance matrix denoted by $\mathbf{\Sigma}_0$ (both estimated with a sample of returns of length k), assuming that returns come from a multivariate normal distribution (with parameters $(\mathbf{X}_0, \mathbf{\Sigma}_0)$), the procedure resamples n times joint returns of length k and estimates different parameters $\{(\mathbf{X}_1, \mathbf{\Sigma}_1), (\mathbf{X}_2, \mathbf{\Sigma}_2), \dots, (\mathbf{X}_n, \mathbf{\Sigma}_n)\}$ which allow us to obtain n efficient frontiers. For a given portfolio, the resampled weights are given by the average of portfolio weights of the n samples:

$$\mathbf{a}_R = \frac{1}{n} \sum_{i=1}^n \mathbf{a}_i, \tag{5}$$

where \mathbf{a}_R is the vector of the asset’s weights in the resampled portfolio, and \mathbf{a}_i ’s are the weights of each of the n realizations.

Several out-of-sample evaluations have shown results in favor of resampling methodology, using different sets of data (see, for instance, Markowitz and Usmen, 2003; Wolf, 2006; Fernandes and Ornelas, 2009). However, these evaluations cannot give definitive conclusions in favor of using resampling, given sampling limitations. Nevertheless, Fernandes and Ornelas (2009) and Kohli (2005) point out that resampled portfolios have two desirable characteristics for long-term investors. First, it usually generates portfolios that have greater diversification as more assets enter into the solution than in classical mean–variance efficient portfolios. Second, the model exhibits smoother transitions and less sudden shifts in allocations as return expectations change, meaning that the transaction costs of rebalancing the portfolio are typically lower. Nevertheless, the traditional resampling methodology, considered as an ad hoc methodology, has been criticized because of its lack of a theoretical basis.

3. Description of the Black–Litterman-resampling methodology

Since the source of estimation error comes at the first part of the BL model and spreads out until the final results, we combine the BL model with resampling techniques. The combined Black–Litterman-Resampling could be summarized as follows:

1. Estimate the BL expected return vector \mathbf{X} and the covariance matrix $\mathbf{\Sigma}$ from historical inputs and maybe also combining with analysts’ views.
2. Resample from results obtained in Step 1, by taking n draws of length L from a multivariate normal distribution with return vector \mathbf{X} and covariance matrix $\mathbf{\Sigma}$.
3. For each draw n , calculate the new expected return and variance matrix. Because estimation error is present, these resampling estimates are different from the ones calculated in Step 1.
4. For each of the n sets of expected returns and covariance matrix calculated in Step 3, calculate the efficient frontier using traditional Markowitz optimization. The output of this step will be a set of n efficient frontiers.
5. For each risk level, calculate the average portfolio weights across the n efficient frontiers. These weights define the portfolios of the BL Resampling frontier. The risk \times return profile of the BLR can be calculated using the expected return vector \mathbf{X} and the covariance matrix $\mathbf{\Sigma}$ from Step 1.

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