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Fuzzy portfolio optimization model under real constraints

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ABSTRACT

This paper discusses a multi-objective portfolio optimization problem for practical portfolio selection in fuzzy environment, in which the return rates and the turnover rates are characterized by fuzzy variables. Based on the possibility theory, fuzzy return and liquidity are quantified by possibilistic mean, and market risk and liquidity risk are measured by lower possibilistic semivariance. Then, two possibilistic mean–semivariance models with real constraints are proposed. To solve the proposed models, a fuzzy multi-objective programming technique is utilized to transform them into corresponding single-objective models and then a genetic algorithm is designed for solution. Finally, a numerical example is given to illustrate the application of our models. Comparative results show that the designed algorithm is effective for solving the proposed models.

1. Introduction

The mean–variance (M–V) model of portfolio selection problem, originally proposed by Markowitz (1952), has played an important role in the development of modern portfolio selection theory. The basic idea of the M–V model is to quantify the expected return of portfolio as the investment risk and use the variance of the expected return of portfolio as the investment risk. After that, variance has been widely used as a risk measure of portfolio such as Yoshimoto (1996), Best and Hlouskova (2000), Xia et al. (2000), Zhang and Nie (2004), Deng et al. (2005), Corazza and Favaretto (2007), Hirschberger et al. (2007), Chen and Huang (2009), etc. However, as pointed out by Grootveld and Hallerbach (1999), the distinguished drawback is that variance treats high returns as equally undesirable as low returns because high returns will also contribute to the extreme of variance. In particular, when probability distribution of security returns are asymmetric, variance becomes a deficient measure of investment risk because it may have a potential danger to sacrifice too much expected return in eliminating both low and high return extremes. To overcome the shortcoming of variance risk measure, some researchers began to employ other risk measures to qualify the risk of portfolio, see for example Markowitz (1959), Konno and Yamazaki (1991), Alexander and Baptista (2004) and Campbell et al. (2001).

As the above literatures mentioned, they often characterized a financial asset as a random variable with a probability distribution over its return. However, in the practical investment, the returns of risky assets are usually in an uncertain economic environment and vary from time to time. So, it is impossible for investors to get the precise probability distributions of the returns on risky assets since there are some other uncertain factors that differ from the random ones affect the financial markets such as economics, policies, laws, regulations, people's psychological factors, etc. In particular, the influence of people's psychological factors cannot be neglected in portfolio decision-making. All these factors mentioned above affect the financial markets such that the returns of risky assets are fuzzy uncertainty. To take human's subjective opinions into consideration, fuzzy approaches are, in general, more appropriate than probabilistic approaches. Thus, it is worthwhile to use fuzzy set theory to investigate the uncertainty in financial markets. Recently, a number of researchers investigated the fuzzy portfolio selection problems. Ammar and Khalifa (2003) proposed a fuzzy portfolio optimization a quadratic programming approach to solve the fuzzy portfolio selection problem. Bilbao-Terol et al. (2006) employed a fuzzy compromise programming technique to deal with the fuzzy portfolio selection problem. Giove et al. (2006) constructed a regret function to solve the interval portfolio selection problem. Chen and Huang (2009) proposed a fuzzy

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optimization model to determine the optimal investment proportion of each cluster. Carlsson and Fullér (2001) reported that the possibilistic mean value and variance concepts of fuzzy number which gain popularity among scholars. After that, many scholars used the possibility measure to investigate fuzzy portfolio selection problems such as Zhang (2007), Zhang et al. (2009) and Carlsson et al. (2002). In addition, some researchers, such as Huang (2008), Qin et al. (2009) and Zhang et al. (2009), Zhang et al. (2010), Zhang et al. (2011), Kamdem et al. (2012) and Gupta et al. (2013) studied portfolio selection problems by using credibility measure.

Though numerous portfolio selection models have been proposed under the framework of fuzzy set theory. Most of them only take the market risk into consideration, namely, the risk of the return on portfolio. To our knowledge, few researchers consider the influence of liquidity risk on portfolio decision-making in fuzzy environment. However, in the real world, the liquidity risk cannot be neglected. If liquidity risk is ignored, it may bring huge economic losses such as British Baring bankruptcy and the "Shan Yi" securities firm in Japan which collapsed. The purpose of this paper is to incorporate both market risk and liquidity risk into the fuzzy portfolio selection problem by taking into account four criteria, including transaction cost, liquidity, transaction lots and cardinality constraint. The contributions of this paper are as follows. We propose two multi-objective fuzzy portfolio optimization models with different cardinality constraints for real portfolio selection. Inspired by Zhang et al. (2012), we design a novel genetic algorithm to solve the proposed model. Besides, we provide a numerical example to illustrate the ideas of the two proposed models and the effectiveness of the designed algorithm.

The rest of this paper is organized as follows. In Section 2, we introduce some basic conceptions of fuzzy numbers. In Section 3, we formulate two types of fuzzy portfolio optimization model with real constraints. In Section 4, we first employ the fuzzy multi-objective programming technique to transform the proposed models into corresponding single-objective models. Then, we design a genetic algorithm to solve the transformed models. In Section 5, we give a numerical example to demonstrate the application of our models and the effectiveness of the designed algorithm. In Section 6, we conclude the paper.

2. Preliminaries

Let us first review some basic concepts about fuzzy number, which we need in the following sections. A fuzzy number A is a fuzzy set of the real line $\mathbb{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by $\mathcal{F}(\mathbb{R})$. A γ-level set of a fuzzy number A is denoted by $[A]^\gamma = \{x \in \mathbb{R} | \mu_A(x) \geq \gamma\}$ (the closure of the support of A) if $\gamma = 0$. It is well known that if A is a fuzzy number then $[A]^\gamma$ is a compact subset on $\mathbb{R}$ for all $\gamma \in [0, 1]$. The above-mentioned definition can be found in Dubois and Prade (1980).

Definition 1 (*Carlsson and Fullér, 2001*)**.** Let A be a fuzzy number with γ-level set $[A]^\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$, $\gamma \in [0, 1]$. Then the possibilistic mean value of A is defined as

$$E(A) = \int_0^1 \gamma(\underline{a}(\gamma) + \overline{a}(\gamma))\mathrm{d}\gamma. \tag{1}$$

Based on Saeidifar and Pasha (2009), if the weighted function $f(\gamma) = 2\gamma$, then the following definitions of the upper and lower possibilistic semivariances of fuzzy number can be obtained as follows.

Definition 2. Let A be a fuzzy number with γ-level set $[A]^\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$, $\gamma \in [0, 1]$, and let $E(A)$ be the possibilistic mean value of A. Then the upper and lower possibilistic semivariances of fuzzy number A are, respectively, defined as

$$\mathrm{Var}^+(A) = \int_0^1 2\gamma(E(A) - \overline{a}(\gamma))^2\mathrm{d}\gamma, \tag{2}$$

$$\mathrm{Var}^-(A) = \int_0^1 2\gamma(E(A) - \underline{a}(\gamma))^2\mathrm{d}\gamma. \tag{3}$$

Definition 3 (*Zhang et al., 2012*)**.** For any two given fuzzy numbers A with $[A]^\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)](\gamma \in [0, 1])$ and B with $[B]^\gamma = [\underline{b}(\gamma), \overline{b}(\gamma)](\gamma \in [0, 1])$, the upper and lower possibilistic semi-covariances between A and B are, respectively, defined as

$$\mathrm{Cov}^+(A, B) = \int_0^1 2\gamma(E(A) - \overline{a}(\gamma))(E(B) - \overline{b}(\gamma))\mathrm{d}\gamma, \tag{4}$$

$$\mathrm{Cov}^-(A, B) = \int_0^1 2\gamma(E(A) - \underline{a}(\gamma))(E(B) - \underline{b}(\gamma))\mathrm{d}\gamma. \tag{5}$$

Based on Zhang et al. (2012), the following results about the upper and lower possibilistic semivariances can be obtained.

Theorem 1. *Let $A_1, A_2, \ldots, A_n$ be n fuzzy numbers, and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be n positive real numbers. Then*

$$\mathrm{Var}^-\left(\sum_{i=1}^n \lambda_i A_i\right) = \sum_{i=1}^n \lambda_i^2 \mathrm{Var}^-(A_i) + 2\sum_{i<j=1}^n \lambda_i\lambda_j \mathrm{Cov}^-(A_i, A_j), \tag{6}$$

$$\mathrm{Var}^+\left(\sum_{i=1}^n \lambda_i A_i\right) = \sum_{i=1}^n \lambda_i^2 \mathrm{Var}^+(A_i) + 2\sum_{i<j=1}^n \lambda_i\lambda_j \mathrm{Cov}^+(A_i, A_j). \tag{7}$$

3. Multi-objective optimization models for fuzzy portfolio selection with real constraints

Assume that there are n risky assets with fuzzy return rates and a risk-free asset with a fixed return rate in financial market for trading. The return rates and turnover rates of the n risky assets are characterized by triangular and trapezoidal fuzzy numbers, respectively. In this paper, the return rate and the turnover rate of risk-free asset are all denoted as constants. For convenience of description, we use the following notations:

- r_i the return rate of risky asset i $(i = 1, 2, \ldots, n)$, where $r_i = (a_i, \alpha_i, \beta_i)$;
- $\widetilde{l}_i$ the turnover rate of risky asset i, where $\widetilde{l}_i = (la_i, lb_i, l\alpha_i, l\beta_i)$;
- r_{n+1} the return rate of risk-free asset $n + 1$;
- x_i the proportion invested in risky asset $i (i = 1, 2, \ldots, n)$ or risk-free asset $n + 1$;
- x_i^0 the initial proportion invested in risky asset $i (i = 1, 2, \ldots, n)$ or risk-free asset $n + 1$;
- c_i the rate of transaction cost for risk asset i $(i = 1, 2, \ldots, n)$;
- u_i the upper bound of the investment proportion x_i $(i = 1, 2, \ldots, n + 1)$.