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Multi-objective genetic algorithms for solving portfolio optimization problems in the electricity market

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ABSTRACT

The multi-objective portfolio optimization problem is not easy to solve because of (i) challenges from the complexity that arises due to conflicting objectives, (ii) high occurrence of non-dominance of solutions based on the dominance relation, and (iii) optimization solutions that often result in under-diversification. This paper experiments the use of multi-objective genetic algorithms (MOGAs), namely, the non-dominated sorting genetic algorithm II (NSGA-II), strength Pareto evolutionary algorithm II (SPEA-II) and newly proposed compressed objective genetic algorithm II (COGA-II) for solving the portfolio optimization problem for a power generation company (GenCo) faced with different trading choices. To avoid under-diversification, an additional objective to enhance the diversification benefit is proposed alongside with the three original objectives of the mean–variance–skewness (MVS) portfolio framework. The results show that MOGAs have made possible the inclusion of the fourth objective within the optimization framework that produces Pareto fronts that also cover those based on the traditional MVS framework, thereby offering better trade-off solutions while promoting investment diversification benefits for power generation companies.

1. Introduction

The Markowitz mean–variance (MV) approach [1] is widely regarded as a ground theory in portfolio selection. This framework assumes that investors make an investment decision in asset allocation in order to maximize their utility by maximizing portfolio return and minimizing portfolio risk subject to a given budget constraint. However, assumptions underlying the MV model such as the quadratic utility function and the normal distribution of returns are often violated, both theoretically [2–4] and empirically [5–7].

In addition, the skewness preference theory and its relevance for applications are widely documented [8–11]. The introduction of skewness in portfolio decision-making brings about a new research direction in portfolio selection. In the mean–variance–skewness (MVS) model [12–14], the mean and skewness of portfolio returns are to be maximized and portfolio risk is to be minimized simultaneously. From the viewpoint of optimization, a solution that simultaneously optimizes all objectives does not exist. Nevertheless a set of compromising solutions can be explored. Besides, the MVS portfolio optimization problem is not easy to solve because the objectives compete and conflict with each other. As a result, the optimal Pareto fronts seem to be non-smooth and discontinuous.

In the literature, both the MV [15,16] and MVS frameworks [17] had been used to set up portfolio optimization problems for electricity generation companies. However, given the fact that electricity spot prices are not normally distributed but skewed, asset allocation based on the MVS framework is more suitable than the MV framework for a generation company (GenCo). In spite of the framework, the shape of the Pareto front presented in previous studies [17], for example, does not reflect the nature of a problem that has competing and conflicting objectives.[1] Further, as observed in previous work [20], the number of assets included in most of portfolio optimization solutions was limited, and had greatly reduced the diversification benefit. In view of the weakness, this paper proposes to include a diversification enhancing objective into the MVS

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[1] The Particle Swamp Optimization (PSO) technique was implemented in [17]. In some problems, GAs can provide better optimal solutions. For instance, the optimal solutions of some well-developed GAs were found to perform better than those obtained through PSO [18,19]. It must be noted that, however, no particular class of multi-objective optimization algorithm consistently outperforms the others in all the application problems.

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portfolio model. Therefore, a four-objective portfolio optimization problem (MVS-D) is formulated for a GenCo that produces and trades electricity in a deregulated electricity market. This inclusion adds to the complexity of the optimization problem by increasing the number of objectives from three to four.

In optimization problems, an increase in the number of conflicting objectives significantly raises the difficulty in the use of an algorithm to find the optimal solution [21]. In conventional multi-objective optimization algorithms (MOOAs), when two candidate solutions are compared, solution **a** does not dominate solution **b** unless all objectives from **a** satisfy the domination condition. With a large number of objectives, the chance that no one solution can dominate the other is expectably high. Therefore, in order for algorithms to provide a good approximation of the true Pareto front, a large number of non-dominated solutions have to be screened using suitable techniques [22,23].

During the past decade, genetic algorithms (GAs) had been successfully applied for solving multi-objective portfolio optimization problems (MOPOPs) in finance subject to different constraints [24,25]. Their applications are also common in multi-objective optimization problems in the power systems [26–29] and other resource allocation problems [30,31]. However, the ability of MOGAs for solving MOPOPs with more than three objectives to be optimized has been rarely investigated. Therefore, the first objective of this paper is to explore if GAs can efficiently and reliably solve MOPOPs with a high number of objectives. The second objective is to conduct a cross-algorithm performance comparison. To achieve these objectives, two well established GAs, namely, non-dominated sorting genetic algorithm II (NSGA-II) [32] and strength Pareto evolutionary algorithm II (SPEA-II) [33], and the newly developed compressed objective genetic algorithm II (COGA-II) [34] were utilized and compared in this study.

The paper is organized as follows. The proposed MOPOP is discussed in Section 2. Section 3 explains the portfolio selection problem in electricity market. A description of the three MOGAs together with the performance comparison criteria are given in Section 4. Section 5 exhibits the numerical experiments and parameter setting. The results and discussions are presented in Section 6, while Section 7 states our conclusions.

2. Multi-objective portfolio optimization model

The notion that investors prefer positive skewness in returns is well documented [8–13]. Therefore, in the mean–variance–skewness (MVS) portfolio optimization problem, expected returns and skewness of portfolio will be maximized, meanwhile, variance of portfolio will be minimized at the same time.

We consider a single decision-making period where N assets are available for investment. At the beginning of the decision making process, investors determine the ratio of their initial wealth to be invested in each available asset. The expected portfolio return, denoted by $R(\mathbf{x}) = \sum_{i=1}^{N} x_i R_i$, where $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is a solution vector, x_i is the ratio of wealth invested in asset i and R_i is the return to asset i which is a random variable that will be realized at the end of the investment period. Portfolio risk is measured by portfolio variance, $V(\mathbf{x})$, while portfolio skewness is denoted by $S(\mathbf{x})$. The MVS portfolio optimization problem can be stated mathematically as follows:

$$\text{Maximize} \quad R(\mathbf{x}) = \sum_{i=1}^{N} x_i R_i \tag{1}$$

$$\text{Minimize} \quad V(\mathbf{x}) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sum_{i=1}^{N} x_i^2 \sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{2}$$

$$\text{Maximize} \quad S(\mathbf{x}) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_i x_j x_k \gamma_{ijk} = \sum_{i=1}^{N} x_i^3 \gamma_i^3 + 3\sum_{i=1}^{N}\left(\sum_{j=1}^{N} x_i^2 x_j \gamma_{iij} + \sum_{j=1}^{N} x_i x_j^2 \gamma_{iij}\right) \tag{3}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1, \quad x_i \geqslant 0 \tag{4}$$

where σ_{ij} is covariance between asset i and j, and γ_{ijk} represents co-skewness between asset i, j and k. The constraint defining the feasible portfolios implies that all capital must be invested in available assets and short sales are not allowed.

The MVS model is a tri-objective optimization problem where objectives are competing and conflicting trade-off exists in different portfolio choices. Procedures and algorithms were introduced in a number of studies for portfolio construction under the MVS framework [12,14,35,36]. These works combined the three objectives to formulate a single objective optimization problem. However, disadvantages of using single objective approach to solve multi-objective optimization problems are widely documented. Since there are three objectives to be optimized, we do not try to find a single optimal solution, but instead a set of optimal solutions, the so called "Pareto-optimal solutions" is searched. According to the Pareto dominance relation, a portfolio $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_N^*)$ is said to be an optimal portfolio if there is no other feasible portfolio $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ such that $R(\mathbf{x}^*) \leqslant R(\mathbf{x})$, $V(\mathbf{x}^*) \geqslant V(\mathbf{x})$ and $S(\mathbf{x}^*) \leqslant S(\mathbf{x})$ with at least one strict inequality.

In addition, as observed by prior study [20], most of the portfolio optimization solutions comprised only a limited number of available assets resulting in the reduction of diversification benefit. In order to avoid excessive investment in a small number of assets, some constraints have been proposed, such as cardinality constraint, ceiling limit constraint and class constraint. However, asset allocation ratios tend to be subjectively determined and it is difficult to identify the ratios without knowing the levels of other objectives [37]. Furthermore, in many cases, investor may not arrive at good solutions if constraint values are forced to be identified beforehand. We handle this problem by minimizing the difference between the highest and the lowest ratio of capital investment in $\mathbf{x}$ as an additional objective in order to enhance diversification benefit. Let this difference be denoted as $D(\mathbf{x})$. Our fourth objective can be stated as follows:

$$\text{Minimize} \quad D(\mathbf{x}) = \max\ \mathbf{x} - \min\ \mathbf{x} \tag{5}$$

In this paper, we formulated the multi-objective portfolio optimization problem denoted by MVS-D as follows:

$$\begin{aligned} &\text{Minimize} \quad F(x) = [-R(\mathbf{x}), V(\mathbf{x}), -S(\mathbf{x}), D(\mathbf{x})] \\ &\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1, \quad x_i \geqslant 0 \end{aligned} \tag{6}$$

3. Portfolio optimization in electricity market

3.1. Trading environment in an electricity market

Similar to financial market, the deregulated electricity market facilitates price efficiency and liquidity by offering various contractual instruments for GenCos to trade their generated products in the different types of electricity markets. This paper considers the situation that a GenCo is making decision to create optimal electricity allocation according to the MVS portfolio model when faced with production capacity constraint. Similar to financial investment, a single investment period is assumed which can be a day, a week, a month and so forth. At the beginning of the decision making period, the GenCo of interest determines the ratio of its initial production capacity to be allocated to available trading