Complex dynamics in an OLG model of neoclassical growth with endogenous retirement age and public pensions

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ABSTRACT

This study analyses the dynamics of a general equilibrium, overlapping-generations (closed) economy with pay-as-you-go public pensions and tax-financed health investments that affect the retirement time when old. Life of the typical agent is divided between youth (firth period) and old age (second period). The latter period of life is, in turn, divided between work time and retirement time in a proportion contingent on an individual's state of health. We show that: (i) a unique non-trivial steady state exists, and (ii) when the labour income tax rates used to finance health expenditure or public pensions are included in an intermediate range of values, complex dynamics occur when individuals have perfect foresight. This holds because the increase either in the fraction of time spent working when old or disability pensions reduces savings and capital accumulation. In addition, dynamic phenomena such as multiple bubbling structures related to the bifurcation diagram can be observed. Under some general assumptions, we show that the rise in health care expenditure and/or public pensions initially destabilises the steady-state equilibrium and causes complex dynamics but eventually acts as a stabilising device.

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1. Introduction

It is recognised that health status of humans and income are significantly correlated and observed to dramatically vary across time and nations (see [1]). The tremendous improvement in lifestyles and standards of living experienced especially in Western countries after World War II, has contributed to make individuals healthier. This fact has accelerated and sustained economic growth in several nations [2], with a resulting increase in both the quantity (longevity) and quality of life of humans, because health spending essentially reduces mortality risks [3,4][3] and fertility, while increasing per capita income [9], even if a non-monotonic relationship that contributes to explain the Demographic Transition is also observed [10–13]. This phenomenon has increased the individual demand for health services, while also requiring the response of governments (especially in developed countries) and international organisations to the question of whether

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   [3] At the time of writing, a lot of people continue to suffer from malnutrition in several developing and underdeveloped countries, where also epidemics (e.g., HIV/AIDS) contribute to keep the individual state of health poorer and child mortality higher (see [5]) than in developed countries, HIV/AIDS being one of the main causes of the reversal in the observed positive trend in life expectancy in such countries (see [6,7]). See also [8], which provides estimates of the impact of the tragedy of AIDS for the South African population.

and to what extent health investments and/or disability pensions have to be financed to an increasing number of old age individuals. Therefore, problems concerning the public provision of health services and pension benefits for mature workers are currently high on the political agenda in several industrialised countries.

The study of problems of the interrelationship between health and macroeconomics – notably, income per person – has gained popularity in both the empirical and theoretical literatures,[4] since the state of health of humans may affect economic behaviours: the role played by health on both the ability to work and productivity of work (see [21]); the relationship between adult mortality and private and/or public health spending (see [1,22–26]); the relationship between adult mortality and the accumulation of human capital due to changes in investments in education (see [27]).

The link between health status and labour productivity has been early recognised in the pioneering paper by Grossman [28]. In addition, several empirical studies have found that health plays a relevant role on the labour supply of older people (e.g. [29–33]). However, to the best of our knowledge, the effects of both public health spending and retirement age on the long-run dynamics in a neoclassical growth model has not been so far investigated in a theoretical context. In this paper we aim at filling this gap by using the overlapping-generations (OLG) model à la Diamond [34], extended with the following assumptions: (i) people inelastically supply labour when old in a proportion contingent on their state of health, (ii) an individual's health status when old is improved by the provision of public health expenditure when young, and (iii) public PAYG pensions exist to support old-age people unable to work and then retired (disability pensions).

In order to concentrate on the effects of health spending and pensions on the length of the age of retirement, we avoid including adult mortality in the analysis and the utility effect of health per se. We also neglect to explicitly account for labour/leisure choices (of both young and old people), which can be an important determinant of macroeconomic outcomes. In particular, we assume that the length of the retirement time depends on an individual's health status when old. This means that the age of retirement is chosen neither voluntarily by mature workers nor it is fixed by the government with appropriate laws. When the health status is low, mature workers are allowed to retire and they are entitled to a pension benefit (alternatively, it may be assumed that for the period of ill-health they receive a pay-as-you-go health insurance bonus because they cannot work).

The main finding of the present paper is the following: when the conventional Diamond's model with rational individuals is extended with the three assumptions above mentioned, the dynamics may be oscillatory. Moreover, both periodic and chaotic dynamics seem to be the rule rather than the exception for this simple one-dimensional OLG economy with logarithmic preferences and Cobb–Douglas technology. So far the OLG economic growth literature has shown that complex dynamics typically occur either in two-dimensional model under rational expectations, when the elasticity of substitution between capital and labour in production is sufficiently small (e.g., the case of Leontief technology),[5] or in one-dimensional models when individual have myopic expectations.[6] In the latter case, periodic and/or complex dynamics occur for high values of the inter-temporal elasticity of substitution in the utility function (e.g., [41–44]). In either cases of rational and myopic expectations, therefore, the OLG economy with logarithmic preferences and Cobb–Douglas technology represents a framework not prone to describe periodic or chaotic dynamic events, even if some exceptions do exist [21,45].

In addition, the long-run dynamics described in the present paper shows a multiplicity of "bubbling" phenomena[7] related to the bifurcation diagram [47], i.e., a sequence of period doubling bifurcations is followed a sequence of period halving bifurcations, when either the pension contribution rate or health tax rate is included in an intermediate range of values. However, further increases in the pension contribution rate or the health tax rate eventually reduce economic fluctuations, thus properly working for the global stability of the economy. This twofold role is remarkable from an economic point of view: on the one hand, it contributes to explain the observed business cycles in per capita income—showing that an endogenous deterministic origin of economic cycles may complement its stochastic origin, the latter being at the core of the real business cycle theory (see [48,49]); on the other hand, the health tax rate may even be used to control and eventually suppress periodic or complex dynamics. Thus, the equilibrium dynamics in this simple economy may reconcile the existence of business cycles [50] or monotonic dynamics [51] depending on the configuration of parameters.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 (resp. 4) analyses the steady-state and dynamic outcomes when the health tax rate (resp. the contribution rate to the pension system) varies. Section 5 concludes.

2. The economy

2.1. Individuals

Consider a general equilibrium OLG closed economy comprised of a continuum of rational and identical individuals of measure one per generation. Population is constant and there is no adult mortality, i.e., an individual is alive at the end of

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[4] On the empirical side, see [14–17]. On the theoretical side, see [18–20].

[5] For instance, Reichlin [35] discusses the Leontief case, while in the Farmer's [36] example with a Constant Elasticity of Substitution (CES) technology, endogenous fluctuations can occur only whether the production function exhibits lower factor substitutability than the Cobb–Douglas function.

[6] Note that interesting economic models either with overlapping generations and discrete time or Ramsey-type models with continuous time exist, where nonlinear dynamics, oscillations and deterministic chaos are studied, especially with regards to problems of environmental sustainability and externality (see [37–40]).

[7] Pioneering discussions about similar dynamic outcomes can be found in [46,47].