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Optimal retirement with unemployment risks ☆

Bong-Gyu Jang *, Seyoung Park, Yuna Rhee

Department of Industrial and Management Engineering, POSTECH, Republic of Korea

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ABSTRACT

This paper investigates the optimal retirement of an individual in the presence of involuntary unemployment risks and borrowing constraints in a complete market with frictions. We use an intensity model and loading factors to illustrate the involuntary unemployment risks and frictions in unemployment insurance markets. Using reasonably calibrated parameters, we observe that high involuntary unemployment intensity and loading factors could be important explanations for the empirical findings emphasized in recent studies. We also find that an individual with high leisure demand after retirement reduces consumption during retirement and increases stockholdings as retirement time approaches.

1. Introduction

During the past decade, a number of researchers have studied optimal choice of retirement time under various conditions, such as uninsurable labor income (Viceira, 2001), an individual's disutility (Choi and Shim, 2006), or flexible labor supply (Bodie et al., 1992). By extending the study of Karatzas and Wang (2000), Farhi and Panageas (2007, henceforth FP) and Dybvig and Liu (2009, henceforth DL) model an individual's labor supply choice as an optimal stopping problem, implying that she can then decide her optimal retirement time voluntarily. However, There are no studies on the optimal retirement time of an individual who encounters unemployment risk (i.e., the risk of involuntary retirement), even though involuntary unemployment is an important social welfare issue. From this perspective, deciding the optimal retirement of an individual given these involuntary unemployment risks plays a key role in policy-making and financial decision making.

This paper investigates an individual's optimal retirement in the presence of involuntary unemployment risks and borrowing constraints in a complete market with frictions. Most existing studies on this optimal retirement problem (e.g., the papers by FP and DL) attempt to find an individual's optimal consumption, investment, and voluntary retirement time simultaneously. Here, we consider another situation in which an individual might be involuntarily unemployed before her optimal retirement date and has borrowing constraints. Moreover, we attempt to construct a complete market with frictions through the purchase of an unemployment insurance policy.

Specifically, this paper makes three major contributions.

First, our approach explicitly considers the role of unemployment risks in a life-cycle model. This life-cycle model is associated with an individual's optimal consumption, portfolio and retirement time, where she is able to choose her retirement time, but could be forced to retire early for various reasons. We include the possibility of an individual's involuntary unemployment into the conventional set-up, and assume that crucial exogenous shocks such as sudden ill-health shocks and layoff shocks cause involuntary unemployment. As far as we know, this is the first study dealing with an individual's optimal retirement policy in the presence of both voluntary retirement and involuntary unemployment possibilities. Our model shows that a larger involuntary unemployment possibility might lead workers to early voluntary retirement, since it restrains the growth of their wealth, and consequently reduces their consumption and stockholdings.

* Corresponding author. Tel.: +82 54 279 2372; fax: +82 54 279 2870.

E-mail addresses: bonggyujang@postech.ac.kr (B.-G. Jang), seyoungdog@postech.ac.kr (S. Park), ynrhee@postech.ac.kr (Y. Rhee).

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Second, we introduce a newly-designed complete market with frictions to solve the optimal retirement problem. Gormley et al. (2010) investigate the role of insurance in the investment and savings decisions of households. The complete market we consider here includes personalized unemployment insurance in which an individual can purchase to hedge her involuntary unemployment risks. However, we assume that the market price of the insurance policy is costly compared with its *actuarially fair* price in order to reflect the consensus that unemployment insurance markets are not competitive relative to other insurance markets. We see this market friction corresponds to *loading* in our model, which is broadly employed terminology in insurance area (Ehrlich and Becker, 1972; Campbell, 1980). We find that increase in the loading factor leads to decrease in consumption and stockholdings.

Finally, we introduce two useful concepts, namely expected time to retirement and the probability of voluntary retirement. We define expected time to retirement as the expectation of the minimum between voluntary retirement time and involuntary unemployment time. We show analytic formulas for both expected time to retirement and the probability of voluntary retirement. To further explore the implications of our model, we use these concepts and illustrate the relationship between expected time to retirement and the life-cycle optimal policies, and the link between changes in the probability of voluntary retirement and an individual's wealth level.

Using our model with realistically calibrated parameters based on related literature dealing with involuntary unemployment (Lachance and Seligman, 2008; Emery et al., 2010), we find some interesting properties, especially related to two factors in our model: involuntary unemployment intensity and fixed rate of the difference between the market price and the *actuarially fair* price of unemployment insurance. The model provides alternative interpretations of the empirical observations regarding the shares of stocks by age in Heaton and Lucas (2000) and the *moderate equity holdings puzzle*, by examining the relationships between the degrees of intensity of involuntary unemployment and optimal policies. Heaton and Lucas (2000) find that people who are close to retirement increase their stockholdings even though traditional portfolio rule states the opposite, and a number of recent studies agree with this finding. The moderate equity holdings puzzle implies that the amount of equity holdings for stock market participants is moderate (Gomes and Michaelides, 2005).

The closest studies to ours are FP and DL. Since the model in this paper determines the optimal retirement time endogenously under the condition that the individual might lose her job, one could say that our paper is simply an extension of the work of FP. Obviously, we add an involuntary unemployment event to the FP model, but adding one more risk to an economic system usually involves some unwanted complexity in solving economic problems. We employ an unemployment insurance, which makes the financial market considered in our model complete.[1] Moreover, we consider borrowing constraints that an individual's wealth level cannot be negative at all times. Even though FP have an analysis with such borrowing constraints, they mention that the borrowing constraints are negligible for high wealth levels and could play a critical role just for wealth levels close to zero. However, as DL model with labor income risks importantly includes borrowing constraints, the borrowing constraints might be important regardless of wealth level in our model, since the absence of such constraints implies that there exist some possibility that an individual enters involuntary unemployment with a negative wealth. We exclude the case of the individual's involuntary unemployment with uncollateralized borrowing.

Although our paper and the paper of DL share some technical approaches, there are several differences. First, our model focuses on both involuntary and voluntary retirement under unemployment risks, whereas the DL model considers voluntary retirement under mortality risks. In our model, the unemployment risks quantified by unemployment intensities have various economic interpretations such as health shocks and layoff shocks, and using such intensities makes our model very tractable. Even though both the unemployment insurance against unemployment risks in this paper and the hedging vehicle (specifically, life insurance against mortality risks or annuity against longevity risks) in the paper by DL are financial tools for market completion, they have a clear difference in terms of time period covered by them. The unemployment insurance can provide its coverage before the voluntary retirement time, while the hedging vehicle in DL provides its coverage depending on the mortality regardless of individual's voluntary retirement time. Second, we take market frictions into consideration and try to find some economic implications by using them. Actually, by investigating various numerical results, we observe that they can have a significant effect on an individual's behaviors in optimum, and we can provide some interesting economic implications. We interpret excess unemployment insurance premiums arising from the market frictions as their loading factors to realistically reflect the fact that unemployment markets are relatively too thin these days.

The paper proceeds as follows. Section 2 specifies our model set-up, and Section 3 describes the analytic solution of our problem. Section 4 introduces useful concepts to be used in proceeding analysis. Sections 5 and 6 present various properties relating to involuntary unemployment intensity and loading factors using empirically plausible parameter values, in the presence and absence of competitiveness in the unemployment insurance market. Finally, Section 7 presents concluding remarks.

2. The model

2.1. Financial market

We start with the conventional set-up with a financial market in the presence of two classes of assets: a risk-free asset (e.g., a bond) and a risky asset (e.g., a stock). The price of a risk-free asset $S_t^{(0)}$ follows

$$dS_t^{(0)} = rS_t^{(0)}dt,$$

where r is a risk-free interest rate. The stock price S_t satisfies

$$dS_t = \mu S_t dt + \sigma S_t dB_t,$$

where μ is the expected rate of the stock return, σ is the stock volatility, B_t is a one-dimensional standard Brownian motion on a suitable probability space $(\Omega, P, \mathcal{F}, \{\mathcal{F}_t\})$. The filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ is the σ-algebra generated by B_t.

Now, we define the *state price density* process (or stochastic discount factor) $H(t)$ as

$$H(t) = \frac{\zeta(t)}{\eta(t)}, \quad H(0) = 1,$$

where $\zeta(t)$ and $\eta(t)$ are

$$\zeta(t) = \exp\left\{-\theta B_t - \tfrac{1}{2}\theta^2 t\right\}, \quad \zeta(0) = 1, \quad \theta = \tfrac{\mu - r}{\sigma},$$
$$\eta(t) = e^{rt}.$$

[1] In general, optimal consumption and investment problems in an incomplete market are hard to solve, and this fact forces us to construct a complete market.