ELSEVIER Journal of Banking & Finance 25 (2001) 1789–1804

Journal of BANKING & FINANCE

www.elsevier.com/locate/econbase

Optimal portfolio selection in a Value-at-Risk framework

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Received 12 November 1999; accepted 24 July 2000

Abstract

In this paper, we develop a portfolio selection model which allocates financial assets by maximising expected return subject to the constraint that the expected maximum loss should meet the Value-at-Risk limits set by the risk manager. Similar to the mean–variance approach a performance index like the Sharpe index is constructed. Furthermore when expected returns are assumed to be normally distributed we show that the model provides almost identical results to the mean–variance approach. We provide an empirical analysis using two risky assets: US stocks and bonds. The results highlight the influence of both non-normal characteristics of the expected return distribution and the length of investment time horizon on the optimal portfolio selection. © 2001 Elsevier Science B.V. All rights reserved.

JEL classification: G11

Keywords: Optimal portfolio selection; Value-at-Risk estimation; Non-normalities; Time horizons

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PII: S0378-4266(00)00160-6

1. Introduction

Modern portfolio theory aims to allocate assets by maximising the expected risk premium per unit of risk. In a mean–variance framework risk is defined in terms of the possible variation of expected portfolio returns. The focus on standard deviation as the appropriate measure for risk implies that investors weigh the probability of negative returns equally against positive returns. However it is a stylised fact that the distribution of many financial return series are non-normal, with skewness and kurtosis pervasive.[1] Furthermore there is ample evidence that agents often treat losses and gains asymmetrically. There is a wealth of experimental evidence for loss aversion (see, for example, Kahneman et al., 1990). The choice therefore of mean–variance efficient portfolios is likely to give rise to an inefficient strategy for optimising expected returns for financial assets whilst minimising risk. It would therefore be more desirable to focus on a measure for risk that is able to incorporate any non-normality in the return distributions of financial assets. Indeed risk measures such as semi-variance were originally constructed in order to measure the negative tail of the distribution separately.

Typically mainstream finance rests on the assumption of normality, so that a move away from the assumption of normally distributed returns is not particularly favoured; one drawback often stated is the loss in the possibility of moving between discrete and continuous time frameworks. However it is precisely this simplifying approach, whereby any deviations from the square root of time rule are ignored, which needs to be incorporated into current finance theory. The ability to focus on additional moments in the return distribution with the possibility of allowing for skewed or leptokurtotic distributions enables additional risk factors (along with the use of standard deviation) to be included into the optimal portfolio selection problem.[2]

In this paper, we develop an optimal portfolio selection model which maximises expected return subject to a downside risk constraint rather than standard deviation alone. In our approach, downside risk is written in terms of portfolio Value-at-Risk (VaR), so that additional risk resulting from any non-normality may be used to estimate the portfolio VaR. This enables a much more generalised framework to be developed, with the distributional assumption most appropriate to the type of financial assets to be employed. We develop a performance index similar to the Sharpe ratio, and for the case that

[1] See among others Fama and Roll (1968), Boothe and Glassman (1987), and Jansen and de Vries (1991).

[2] Recent research by Harvey and Siddique (2000), Bekaert et al. (1998) and Das and Uppal (1999) indeed advocate the need to incorporate non-normalities into the portfolio allocation decision.