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On optimal portfolio choice under stochastic interest rates☆

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Abstract

In an economy where interest rates and stock price changes follow fairly general stochastic processes, we analyze the portfolio problem of an investor endowed with a non-traded cash bond position. He can trade on stocks, the riskless asset and a futures contract written on the bond so as to maximize the expected utility of his terminal wealth. When the investment opportunity set is driven by an arbitrary number of state variables, the optimal portfolio strategy is known to contain a pure, preference free, hedge component, a speculative element and Merton–Breeden hedging terms against the fluctuations of each and every state variable. While the first two components are well identified and easy to work out, the implementation of the last ones is problematic as the investor must identify all the relevant state variables and estimate their distribution characteristics. Using the martingale approach, we show that the optimal strategy can be simplified to include, in addition to the pure hedge and speculative components, only two Merton–Breeden-type hedging elements, however large is the number of state variables. The first one is associated with interest rate risk and the second one with the risk brought about by the co-movements of the spot interest rate and the market prices of risk. The implementation of the optimal strategy is thus much easier, as it involves estimating the

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characteristics of the yield curve and the market prices of risk only rather than those of numerous (a priori unknown) state variables. Moreover, the investor's horizon is shown explicitly to play a crucial role in the optimal strategy design, in sharp contrast with the traditional decomposition. Finally, the role of interest rate risk in actual portfolio risk management is emphasized.

JEL classification: G11; G13

Keywords: Portfolio choice theory; Hedging; Interest rate risk; Financial futures; Market price of risk

1. Introduction

The overall outburst of interest rate volatility that has plagued recurrently most Western and South-East Asian economies since the late 1970s accelerated the need for and the creation of new speculative and hedging instruments, such as swaps and derivatives. It has also elicited important developments in investment concepts and techniques. This paper examines the issue of optimal portfolio policy in a multi-period model where investors maximizing expected utility of terminal wealth face, in particular, interest rate risk. More precisely, it offers to contribute to the investment and hedging problem in the rather general case where the value of traded and non-traded assets depends, more or less crucially, on the stochastic behavior of interest rates.[1] For instance, all financial institutions, most non-financial firms and individual investors do face this situation.

In an economy where interest rates and stock price changes follow fairly general stochastic processes, we thus analyze the portfolio problem of an investor endowed with a non-traded cash bond position. He can trade on stocks, the riskless asset and a futures contract written on the bond so as to maximize the expected utility of his terminal wealth. The drift and diffusion parameters of all involved stochastic processes are driven by an arbitrary number of state variables, so that investors face a stochastic investment opportunity set. The traditional route followed in the literature is to use the stochastic dynamic programming technique, leading to the Hamilton–Jacobi–Bellman equation. In such a context, the investor's optimal portfolio strategy is known to contain a pure, preference free, hedge component, a speculative element and Merton–Breeden hedging terms against the random fluctuations of each and every

[1] The bulk of the literature related to optimal dynamic hedging (through futures or forward contracts) assumes either that the cash position to be hedged is not sensitive to interest rate risk or that interest rates are deterministic, or both. See, for example, Adler and Detemple (1988), Duffie and Jackson (1990), Duffie and Richardson (1991) and Lioui and Poncet (1996).