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Intervention options in life insurance

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Abstract

We deal with the intervention options of the policy holder in life insurance. To these options belong the surrender and the free policy (paid-up policy) options. Our approach is to let payments be driven by processes in which the policy holder is allowed to intervene. The main result is a quasi-variational inequality describing the market reserve on an insurance contract taking into account intervention options. The quasi-variational inequality generalizes Thiele's differential equation used for calculation of reserves on a policy without intervention options. It also generalizes the classical variational inequality used for calculation of the price of an American option.

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1. Introduction

The market reserve on an insurance contract is in Steffensen (2000b) defined as the market price of future contractual payments. We shall work with the same notion of the reserve, but since this is the only reserve present here we will suppress the word market and simply speak of the reserve. It is a difficult task to describe in detail the payments stipulated in an insurance contract including the various options that may be held by the policy holder, the insurance company, and the supervisory authorities. In several papers, published during the last decades, the authors bring some of these options to the surface and deal with their impact on the pricing and the reservation problems.

Starting with unit-linked life insurance, Brennan and Schwartz (1976) recognized the option structure of a unit-linked life insurance contract with a guarantee. Brennan and Schwartz integrate the mathematics of finance inevitably as a part of the mathematics of insurance. Going to participating life insurance, the application of mathematical finance has been long in coming probably due to the complex nature of these products. Briys and de Varenne (1994) made the first attempt at dealing with the bonus option of the policy holder and the bankruptcy option of the (owners of the) insurance company in terms of contingent claims analysis. Since then, the idea has been developed in various respects. Miltersen and Persson (1998) deal with the bonus option, whereas Grosen and

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Jørgensen (2000a) in addition take into consideration the surrender option. Grosen and Jørgensen (2000b) formalize the bankruptcy option of the insurance company and the intervention option of the supervising authorities. Important references for the point of view taken in the present paper are Grosen and Jørgensen (1997, 2000a), connecting early exercise with the surrender option in life insurance.

We deal with the type of options that can be described by control by intervention. With control by intervention is meant that the controller is allowed to intervene in the evolution of some specified controlled process by introducing a jump at some controller-specified stopping time in the sense that he can actively move the process to some new point in the state space. The simplest form of intervention is optimal stopping, and like Grosen and Jørgensen (1997) we shall connect theory of optimal stopping with the surrender option in life insurance. The surrender option is the primary intervention option, often held by the policy holder, but there may be others. An example is the free policy option where the policy holder can, at any point in time, stop the payment of premiums but continue the contract with subscribed benefits.

In life insurance mathematics based on finite-state Markov processes, the classical approach to the surrender option in life insurance is to incorporate a state and an intensity of surrender (see, e.g. Hoem, 1988). It is, however, important to realize that the transition to the surrender state happens on demand of the policy holder and not by accident as does the transition to, e.g. the state of death. This indicates that these transitions should be handled differently. The classical approach may, in fact, be dangerous in connection with market valuation where some conditions in the market may urge the policy holders to stay in the contracts whereas other conditions may urge them to surrender. Our approach to valuation of intervention options is based on optimal intervention strategies giving rise to arbitrage free values.

The intervention options described above belong exclusively to the policy holder. However, also the insurance company may hold intervention options, and the bankruptcy option is the primary example. Allowing for intervention options of both the policy holder and the insurance company, we can regard the insurance contract as a so-called game option (see Kifer, 2000). However, we shall not pursue this idea in this paper.

Due to the fact that most of the derivatives traded on the financial markets in practice are of American type, i.e. derivatives including an early exercise option, theory of optimal stopping plays an important role in derivative pricing. Many textbooks on mathematical finance contain an introduction (see, e.g. Lamberton and Lapeyre, 1996).

A main result in optimal stopping is the expression of the optimal value function in terms of a solution to a so-called variational inequality. Description of the solution in the case of the American option were studied in, e.g. Jaillet et al. (1990). The main result of the present paper is a quasi-variational inequality, the solution of which expresses the reserve on an insurance contract with intervention options held by the policy holder. Since the American option is a special case of the setup of payments in this paper, we choose to repeat, in a separate section, the well-known result for this case. On one hand, this will give the reader with a background in insurance but without particular knowledge to American option pricing an introduction to this well-studied object in the literature on mathematical finance. On the other hand, this will help the reader with a background in finance but without particular knowledge to insurance options to comprehend the setup introduced here.

In Section 2, we present the stochastic environment and introduce the payment process that makes up our insurance contract. In Section 3, the main results are presented in two theorems. In Sections 4–6, we illustrate the main results in the cases of the American option in finance, the surrender option in life insurance, and the free policy option in life insurance, respectively.

2. The environment

In this section we recapitulate and extend the framework developed in Steffensen (2000b), which corresponds exactly to the special case of no intervention. We ask the reader to confer Steffensen (2000b) for motivation, details, and examples in this special framework, and we give here only interpretations and comments on the inclusion of intervention options.