Available online at www.sciencedirect.com

SCIENCE DIRECT®

Insurance: Mathematics and Economics 35 (2004) 113–136

INSURANCE
MATHEMATICS & ECONOMICS

www.elsevier.com/locate/econbase

Stochastic mortality in life insurance: market reserves and mortality-linked insurance contracts

Mikkel Dahl

Laboratory of Actuarial Mathematics, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

Abstract

In life insurance, actuaries have traditionally calculated premiums and reserves using a deterministic mortality intensity, which is a function of the age of the insured only. Here, we model the mortality intensity as a stochastic process. This allows us to capture two important features of the mortality intensity: Time dependency and uncertainty of the future development. The advantage of introducing a stochastic mortality intensity is twofold. Firstly, it gives more realistic premiums and reserves, and secondly, it quantifies the risk of the insurance companies associated with the underlying mortality intensity. Having introduced a stochastic mortality intensity, we study possible ways of transferring the systematic mortality risk to other parties. One possibility is to introduce mortality-linked insurance contracts. Here the premiums and/or benefits are linked to the development of the mortality intensity, thereby transferring the systematic mortality risk to the insured. Alternatively, the insurance company can transfer some or all of the systematic mortality risk to agents in the financial market by trading derivatives depending on the mortality intensity.

JEL classification: G22

Subj. class: IM40; IE50; IB10

Keywords: Stochastic mortality intensity; Systematic mortality risk; Unsystematic mortality risk; Incomplete markets; Derivative pricing

1. Introduction

Traditionally, actuaries have been calculating premiums and reserves using a deterministic mortality intensity, which is a function of the age only, and a constant interest rate (representing the payoff of the investments made by the companies). However, since neither the interest rate nor the mortality intensity is deterministic, life insurance companies are essentially exposed to three types of risk when issuing contracts: Financial risk, systematic mortality risk and unsystematic mortality risk. Here, we distinguish between systematic mortality risk, referring to the future development of the underlying mortality intensity, and unsystematic mortality risk, referring to a possible adverse development of the insured portfolio. So far the life insurance companies have dealt with the financial and (systematic) mortality risks by choosing both the interest rate and the mortality intensity to the safe side, as seen from the insurers' point of view. When the real mortality intensity and investment payoff are experienced over time, this usually leads to a surplus, which, by the so-called contribution principle, must be redistributed among

E-mail address: dahl@math.ku.dk (M. Dahl).

0167-6687/$ – see front matter
doi:10.1016/j.insmatheco.2004.05.003

the insured as bonus, see Norberg (1999). Since insurance contracts often run for 30 years or more, a mortality intensity or interest rate, which seems to be to the safe side at the beginning of the contract, might turn out not to be so. This phenomenon has in particular been observed for the interest rate during recent years, where we have experienced large drops in stock prices and low returns on bonds. However, the systematic mortality risk is of a different character than the financial risk. While the assets on the financial market are very volatile, changes in the mortality intensity seem to occur more slowly. Thus, the financial market poses an immediate problem, whereas the level of the mortality intensity poses a more long term, but also more permanent, problem. This difference could be the reason why emphasis so far has been on the financial markets. We hope to turn some of this attention towards the uncertainty associated with the mortality intensity by modelling it as a stochastic process.

In order to obtain a more accurate description of the liabilities of life insurance companies, market reserves have been introduced, see Steffensen (2000) and references therein. Here, the financial uncertainty as well as the uncertainty stemming from the development of an insurance portfolio with known mortality intensity is considered. By modelling the mortality intensity as a stochastic process, market reserves can be further extended to include the uncertainty associated with the future development of the mortality intensity. This should allow for an even more accurate assessment of future liabilities, since possible trends in the mortality intensity and the market attitude towards systematic mortality risk can be taken into account. In addition, a stochastic mortality intensity allows for a quantification of the systematic mortality risk of the insurance companies. Having quantified the systematic mortality risk, we investigate how the insurance companies could manage the risk. As a first possibility, we introduce a new type of contracts called *mortality-linked contracts*. The basic idea is to link and currently adapt benefits (and/or premiums) to the development of the mortality intensity in general, and thereby transfer the systematic mortality risk from the insurance company to the group of insured. A second possibility is to transfer the systematic mortality risk to other parties in the financial market. Here, the idea is to introduce certain traded assets, which depend on the development of the mortality intensity.

This paper is organized as follows: Section 2 contains a review of existing literature on stochastic mortality. Section 3 deals with the modelling of the mortality intensity as a stochastic process, and Section 4 introduces the model considered in the rest of this paper. An expression for the market reserve for a general payment stream is given in Section 5. In Section 6, we introduce the concept of a mortality-linked insurance contracts. Finally, Section 7 includes a discussion of how the systematic mortality risk could be transferred to other agents in the financial market.

2. Existing literature on stochastic mortality

In this section we give a brief review on existing literature concerning the uncertainty associated with the future development of the mortality intensity. For further references see the referred papers.

Olivieri (2001) assumes that the insurance companies take possible trends in the mortality intensity into account by estimating a mortality intensity, which is a function of both time and age. Hence the companies obtain more realistic premiums and reserves than by using a function of age only. However, the estimated survival function, no matter how good it is, is only one possible future development. Thus, Olivieri uses the observed mortality intensities to generate two additional survival functions, which represent very high and very low future survival probabilities, respectively. Using these three possible scenarios for the future survival function, Olivieri illustrates the impact of systematic mortality risk by calculating variances of present values. Marocco and Pitacco (1998) model the yearly mortality rates via a beta distribution with age- and time-dependent parameters. Hence they are able to quantify the mortality risk inherent in an insurance portfolio, since the number of survivors follows a binomial-beta distribution. The approach in Olivieri and Pitacco (2002) is somewhat different. They describe the future survival function by a parameterized family of possible future survival functions. However, since the future is unknown, the parameter is a random variable. In order to obtain prices and assess the risk they apply Baysian methods to describe the distribution function for the parameter. Within this model they are able to distinguish between the unsystematic