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Risk measure and fair valuation of an investment guarantee in life insurance

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Abstract

Investment guarantees are amongst the most important topics in the pricing and management of life insurance. Traditionally, two ways of analyzing the risk are possible: on the one hand, the financial approach based on risk-neutral measure and leading to option pricing and continuous hedging strategy and on the other hand, a more actuarial approach based on ruin probability and distribution of surplus. The purpose of this paper is to try to integrate these two approaches in the management of life insurance contracts with profits. First, we analyze in terms of value at risk and conditional value at risk the effect of putting an investment guarantee. This will be done in an ALM framework, based on different investment strategies of the insurer in terms of risk and matching between assets and liabilities. The liability side will be represented by a guaranteed technical rate; the asset side will be a mix of stocks, cash and bonds in a Gaussian environment with different matching strategies. Consequences of an investment choice in terms of ruin probability and level of solvency will be illustrated. In a second step, fair valuation principles are used in order to compute the market value of the contract and fix the participation rate of the contract.

Keywords: Guaranteed rate; Participation rate; Risk neutral pricing; ALM; Value at risk; Expected shortfall

1. Introduction

Investment guarantees embedded in classical life insurance contracts are surely nowadays one of the most important challenges for the insurance industry. In order to cope with this risk, two big paradigms seem to be available for the actuary.

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1.1. The risk-neutral approach

Under classical assumptions on the market (completeness, no arbitrage, . . .) the price of the guarantee is associated to the price of an option and the machinery of option pricing can be used in order to valuate the product. This philosophy has been used with success as well for equity linked contracts with maturity guarantee (see, for instance, Brennan and Schwartz, 1976; Delbaen, 1986; Aase and Persson, 1994; Nielsen and Sandmann, 1995) as for life insurances with profit (see, for instance, Bacinello, 2001, 2003; Grosen and Jorgensen, 2000). In this context, computations take place under a risk-neutral probability measure. We can consider these models as directly inspired by modern finance theory.

1.2. The risk-management approach

Modern tools of simulation permit more and more to generate a lot of future scenarios and to build the distribution of the final surplus of the insurer taken into account stochastic financial models. Parameters of the contracts are then fixed in relation with a certain level of solvency. Value at risk concept or more generally risk measures are the central tools (Artzner et al., 1997, 1999; Wirch and Hardy, 1999). Of course, computations are done in the real historical world. These models directly linked with dynamic financial analysis (DFA, developed as well in life as in non life insurance-see, for instance, Kaufmann et al., 2001) are in fact close to the classical risk theory and the actuarial well-known concept of probability of ruin.

These two methodologies have already been compared in terms of reserving (Boyle and Hardy, 1997). The purpose of this paper is to propose a way to combine these two methodologies in order to fix the technical parameters of a classical life insurance product with profit. In this context we suppose that the insurer guarantees a certain fixed rate on the paid premiums. On top of that, participation is given at maturity if the real investment performances are good. This bonus is expressed as a rate applied to the eventual final surplus of the insurer.

So, the decision problems for the insurer in this kind of product consists of choosing two numbers: the *guaranteed rate* and the *participation rate*. In order to proceed, we propose to decompose the problems in two steps:

First step: In our mind, the choice of a guaranteed rate is directly linked with solvency concerns. Perfect hedging for long periods as in life or pensions contracts is a utopia. Derivative instruments for very long periods as in life or in pension insurance are not common on the markets and self-hedging is too much expensive.

A risk-neutral price could give the illusion of an absence of risk like in the Black and Scholes world but in fact risk is still there at maturity if you do not hedge perfectly as requested by the underlying pricing process. All this motivates to use a risk-management approach in order to fix the guaranteed rate for long periods.

Second step: Once this technical rate is chosen and in order to fix the second parameter – the participation rate – we can try to build a contract equilibrated between the policyholder and the insurer. Risk-neutral fairness is then used to compute an equilibrated value for the participation rate.

By this way, combination of the two paradigms permits to take into account at the same time as well solvency concerns as fair valuation principles.

The paper is organized as follows. Section 2 presents the main assumptions on the financial market and the investment strategies. In this context we develop a model with cash, bonds and stocks (cash–bond–stock model, CBS model hereafter) representing better than other simple models (for instance, binomial or geometric Brownian) the fundamental investments used by the insurers as underlying values for life insurance products with profit. Different kinds of bond strategies are especially developed in order to model the influence of matching policies.

Section 3 is then devoted to the risk-management analysis in order to select the rate that can be guaranteed. Explicit solutions in the CBS model are given using a “value at risk” approach; not surprisingly the rate is a function of the required level of solvency. Formulas are also given if we choose as criteria the expected shortfall instead of the value at risk.