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A Lévy process-based framework for the fair valuation of participating life insurance contracts

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Abstract

In this communication, we develop suitable valuation techniques for a with-profit/unitized with profit life insurance policy providing interest rate guarantees, when a jump-diffusion process for the evolution of the underlying reference portfolio is used. Particular attention is given to the mispricing generated by the misspecification of a jump-diffusion process for the underlying asset as a pure diffusion process, and to which extent this mispricing affects the profitability and the solvency of the life insurance company issuing these contracts.

JEL classification: G13; G23

MSC: IB10, IM10

Keywords: Esscher transform; Fair value; Incomplete markets; Lévy processes; Participating contracts

1. Introduction

Participating life insurance policies are investment/saving plans or contracts (with associated life insurance benefits) which specify a benchmark return, an annual minimum rate of return guarantee and a surplus distribution mechanism, that is, a rule for the distribution of the annual investment return in excess of the guaranteed return between the insurer and the customer. These contracts make up a significant part of the life insurance market of many industrialised countries including the US, Canada, Japan and members of the European Union.

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These kinds of contract represent liabilities to the issuers implying that their value and the potential risk to the insurance company's solvency should be properly valued. To the extent that, as a result of the difficulties that un-hedged guarantees embedded in these contracts have caused to the life insurance industry in recent years, the regulatory authorities have increased the monitoring of insurance companies' exposure to market risk, credit risk and persistency risk induced by participating contracts, and the embedded options included in the design of these contracts. For example, in the UK, the potential threat to the company's solvency from with-profit policies has been addressed by the Financial Services Authority (FSA) with the introduction, into the regulatory regime for life insurance companies, of the twin peaks approach for the assessment of the financial resources needed for with-profit business. Such an approach (as described in CP195) requires the insurer to set up realistic balance sheets that are designed to capture the cost of guarantees and smoothing on a market consistent basis, so that the firm's provisions are more responsive to changes in the market value of the backing assets for the with-profit funds. This implies the implementation of adequate, consistent and objective models for both the behaviour of the price of the assets backing the policy and the calculation of realistic liabilities, where by liability it is meant all of the guaranteed elements in the policy plus the projection of future discretionary bonus payments. The development of these market-oriented accounting principles for insurance liabilities reflects the more general recommendations from the International Accounting Standards Board (IASB) accounting project (known as International Financial Reporting Standards, or IFRS), and the EU Solvency II review of insurance firm's capital requirements. IFRS will become particularly important as, from 2005, essentially all the EU companies that are listed on European exchanges will be required to produce balance sheets in accordance with IFRS.

In light of the international move promoted by IASB towards the market-based, fair value accountancy standards mentioned above, in this paper we apply classical contingent claim theory for the valuation of the most common policy design used in the UK for participating contracts. In fact, since the pioneering work of Brennan and Schwartz (1976) on unit-linked policies, there have been several studies on the different typologies of contract design and their features. Thus, we would cite Bacinello (2001, 2003), Ballotta et al. (2004), Grosen and Jørgensen (2000, 2002), Guillén et al. (2004), Haberman et al. (2003) and the references therein, and Tanskanen and Lukkarinen (2003), just to mention some of the most recent works.

It is worth pointing out that all these contributions use a Black–Scholes (1973) framework, based on the assumption of a geometric Brownian motion model for the dynamics of the asset fund backing the insurance policy. However, the dramatic changes shown by financial markets over the last 15 years suggest that a better specification of this underlying temporal evolution is needed. In particular, the evidence suggests very strongly that log-stock returns have fatter tails than the normal distribution, meaning that the normal distribution understates the probability of extremes events, especially falls, in the stock prices, thereby inducing biases in the option prices. Alternative models for the asset's return process have been investigated since the early 1960s, for example, by Mandelbrot (1963) and Fama (1965). Extensions to the Black–Scholes model for option pricing began appearing in the finance literature not long after publication of the original paper in 1973. For example, Merton (1973) generalized the Black–Scholes formula to account for a deterministic time-dependent rather than constant volatility later in the same year and, in 1976, he incorporated jump-diffusion models for the price of the underlying asset. From those seminal works, a vast literature on generalizations of the model arose; a state-of-the-art evaluation and comparison of some of these models is contained, for example, in Bakshi et al. (1997).

The purpose of this communication is to consider the valuation problem for one of the smoothing schemes commonly used by insurance companies in the UK and analyzed by Haberman et al. (2003), when a more realistic formulation of the stochastic process driving the reference portfolio is made, than the usual geometric Brownian motion. In particular, we set up a market model based on the use of a Lévy motion as relevant process for the value of the underlying reference portfolio's returns. In this framework, we consider the problem of determining the fair value of a with profit policy in which the reversionary bonus rate is based on the idea, widely adopted in the UK, of a smoothed "asset share" scheme (Needleman and Roff, 1995).

The rest of the paper proceeds as follows: in Section 2, we introduce the participating policy under consideration and the details of the benefits it offers; in Section 3, we develop the market setup and the model for the valuation