Hedging life insurance contracts in a Lévy process financial market

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Abstract

Starting from the model of Møller (Risk-minimizing hedging strategies for unit-linked life insurance contracts. ASTIN Bulletin 28 (1998) 17–47) we derive analogously, but for an incomplete financial market, a (locally) risk-minimizing hedging strategy for unit-linked life insurance contracts represented by the pure endowment and the term insurance. The incomplete financial market is exemplarily given by a general Lévy-driven model. We investigate the Föllmer–Schweizer decomposition of their intrinsic value. Additionally, we compare our results to the ones obtained by Møller (Risk-minimizing hedging strategies for unit-linked life insurance contracts. ASTIN Bulletin 28 (1998) 17–47) and show how they are affected by replacing the complete financial market by an incomplete one.

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1. Introduction

The paper studies the model of Møller (1998) and demonstrates how his results are affected when the complete Black–Scholes market is replaced by an incomplete financial market in case of a more general geometric Lévy-driven model as it was introduced by Chan (1999). In detail, we derive hedging strategies of unit-linked life insurances in this incomplete financial market whereas we follow Møller (1998) in considering a pure endowment and a term insurance. The idea of Møller (1998) is to model the uncertainty of both the insured lives and the stock price simultaneously as a product probability space and not to average away mortality like it is the case with the actuarial principle of equivalence. With our modification the intrinsic risk of the insurance company arises now not only from the insurance portfolio but also from the financial market. This risk and the hedging strategies are derived with the (local) risk-minimization theory of Föllmer and Sondermann (1986) and Schweizer (1991), assuming that payable insurance benefits are compounded risk-free and cashed out at the end of the considered time horizon. Premiums are supposed to be payed as single premium at the beginning. For an extension, allowing intermediate payment times, we refer to Møller (2001), where...
the martingale case is treated. The more general semimartingale case is currently being worked out. For the pure endowment we discuss and compare in detail our results with the results of Möller (1998) and show that the financial risk is not diversifiable by raising the number of insured persons within the portfolio. This is however the case with risk originated by mortality. For completeness we simply state the result for the term insurance, for which the same reasoning like for the pure endowment applies.

2. The model

Our model builds on \((\Omega_2, F, (\mathcal{F}_t)_{t \geq 0}, P_2)\), which is a product space of two independent probability spaces. The first, \((\Omega_1, \mathcal{F}, (\mathcal{G}_t)_{t \geq 0}, P)\), represents a Lévy process financial market and the second, \((\Omega_2, \mathcal{H}, (\mathcal{H}_t)_{t \geq 0}, P_2)\), is used to describe an insurance portfolio. The time horizon of both models is a finite positive number \(T \in \mathbb{R}\). We assume that the mentioned probability spaces satisfy the usual hypothesis of right continuity and completeness.

2.1. The insurance portfolio

The insurance portfolio \((\Omega_2, \mathcal{H}, (\mathcal{H}_t)_{t \geq 0}, P_2)\) is like in Möller (1998). One considers \(l_x\) individuals, all of equal age \(x\), with i.i.d. non-negative lifetimes \(T_1, \ldots, T_{l_x}\). With their hazard rate \(\mu_{x+t}\), the survival function is given by

\[ s_{x+t} = \mathbb{P}(T_1 > t + x | T_1 > x) = \exp \left( - \int_0^t \mu_{x+r} \, dr \right). \]

The number of deaths in this group is modeled by the process \(\sum_{i=0}^{l_x} \mathbb{1}(T_i \leq t), \quad 0 \leq t \leq T.\) The filtration \((\mathcal{H}_t)_{t \geq 0}\) is assumed to be the natural filtration generated by \(\sum_{i=0}^{l_x} \mathbb{1}(T_i \leq t), \quad 0 \leq t \leq T.\) We assume further that the Lévy measure \(\mu_{x+t}\) satisfies:

\[ \int_0^\infty \mathbb{1}(x \geq t) \, dx \times \langle M \rangle_t = \int_0^t \lambda_u \, du, \quad \text{where} \quad \lambda_u = (N_u^1 - N_u^0) \mu_{x+u}, \]

defines a martingale with \(\langle M \rangle_t = \int_0^t \lambda_u \, du, \quad 0 \leq t \leq T.\) As in Möller (1998) we follow Aase and Persson (1994) in supposing risk-neutrality of an insurer towards mortality, i.e. that \(P_2\) is already the risk neutral measure.

2.2. The financial market

The Lévy process financial market \((\Omega_1, \mathcal{G}, (\mathcal{G}_t)_{t \geq 0}, P)\) was introduced by Chan (1999). For a reference on Lévy processes we refer to e.g. Protter (2004) (Chapter I, Section 4) or Jacod and Shiryaev (2003) (Chapter II). Let \((\mathcal{L}_t)_{t \geq 0}\) with \(\mathcal{L}_0 = 0\) a.s. be the càdlàg version of a Lévy process. The filtration \((\mathcal{G}_t)_{t \geq 0}\) is supposed to be the natural filtration of \(\mathcal{L}_t\). \(\mathcal{G}_0\) is trivial and \(\mathcal{G}_T = \mathcal{G}\). We assume further that the Lévy measure \(\nu(dx)\) of \(\mathcal{L}_t\) satisfies:

\[ \int_0^\infty \mathbb{1}(x \geq t) \, dx \times \langle \mathcal{M} \rangle_t = \infty. \]

This allows one to decompose (cf. Chan, 1999) the process \(\mathcal{L}_t\) into

\[ \mathcal{L}_t = c \mathcal{W}_t + \mathcal{M}_t + at, \quad 0 \leq t \leq T, \]

where \((c \mathcal{W}_t)_{t \geq 0}\) is a Brownian motion with standard deviation \(c > 0\), \(a = \mathbb{E}(\mathcal{L}_t)\) and

\[ \mathcal{M}_t = \int_0^t \int \mathbb{1} s \, (dx, \, ds), \quad 0 \leq t \leq T, \]

is a square-integrable martingale. The measure \(M(dx, ds)\) denotes the compensated Poisson random measure on \([0, \infty) \times \mathbb{R} \setminus \{0\}\) corresponding to the jumps of \(\mathcal{L}_t\). For notational convenience we set \(M(dx, \{0\}) = \nu(dx) = 0.\)
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