Risk-neutral valuation of participating life insurance contracts

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Abstract

The valuation of life insurance contracts using concepts from financial mathematics has recently attracted considerable interest in academia as well as among practitioners. In this paper, we will investigate the valuation of participating contracts, which are characterized by embedded interest rate guarantees and some bonus distribution rules. We will model these under the specific regulatory framework in Germany; however, our analysis can be applied to any insurance market with cliquet-style guarantees.

We will present a framework, in which different kinds of guarantees or options can be analyzed separately. Also, the practical implementation of such models is discussed. We use two different numerical approaches to derive fair parameter settings of such contracts and price the embedded options.

The sensitivity of the contract value with respect to multiple parameters is studied. In particular, we find that life insurers offer interest rate guarantees below their risk-neutral value. Furthermore, the financial strength of an insurance company considerably affects the value of a contract.

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1. Introduction

Participating life insurance policies often contain an interest rate guarantee. In many products, this guarantee is given on a point-to-point basis, i.e. the guarantee is only relevant at maturity of the contract and not increased by bonus distribution during the term of the contract. In other products (which are predominant e.g. in the German market), there is a so-called cliquet-style guarantee. This means that the policy holders have an account to which each year a certain rate of return has to be credited. Usually, the life insurance companies provide the guaranteed rate of interest plus some surplus on the policy holders’ account every year. Considering the big market share of such

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products in many countries, the analysis of life insurance contracts with a cliquet-style guarantee is very important. However, a comparatively large portion of the academic literature focuses on guarantees in unit-linked, equity-linked or variable life insurance contracts (e.g. Brennan and Schwartz (1976) or Aase and Persson (1994)).

The analysis of participating policies with a cliquet style guarantee requires a realistic model of bonus payments. For instance the approach presented in Grosen and Jørgensen (2000) explicitly models a bonus account, which permits smoothing the reserve-dependent bonus payments. Smoothing the returns is often referred to as the “average interest principle”. Aside from the guarantee and a distribution mechanism for excessive returns, Grosen and Jørgensen’s model includes the option for the policy holder to surrender and “walk away”. In this case, the policy holder obtains his account value whereas the reserves remain with the company. Since the account value is path dependent, they are not able to present closed form solutions for the risk-neutral value of the liabilities. Monte Carlo methods are used for the valuation and the analysis.

Similarly, in Miltersen and Persson (2003) a cliquet-style guarantee, a bonus account and a distribution mechanism are considered. Here the return exceeding the guaranteed level is distributed between the policy holders’ account, the company’s account and an account for terminal bonus. If in some year the return on assets is below the guaranteed rate, the bonus account can be used to fulfill the guarantee. In particular, the bonus account can become negative, but the insurer has to consolidate a negative balance at the end of the insurance period. However, a positive balance is completely credited to the policy holders.

In Hansen and Miltersen (2002), a hybrid of the models by Miltersen and Persson (2003) and Grosen and Jørgensen (2000) is presented. They use the same model for the distribution mechanism as in Grosen and Jørgensen (2000), but the account structure from Miltersen and Persson (2003). Besides a variety of numerical results, they focus on the analysis of the “pooling effect”, i.e. they analyze the consequence of pooling the undistributed surplus over two inhomogeneous customers.

In Bacinello (2001) and Bacinello (2003), also cliquet-style guarantees are considered. In Bacinello (2001), she prices participating insurance contracts with a guaranteed interest rate in a Black–Scholes market model. Here, as in Miltersen and Persson (2003), the bonus is modeled as a fixed fraction of the excessive return. She finds closed form solutions for the prices of various policies. In Bacinello (2003), she additionally allows for the surrender of the policy and presents numerical results in a Cox–Ross–Rubinstein framework.

Grosen et al. (2001) introduce a different numerical approach to their valuation problem from Grosen and Jørgensen (2000) using the Black–Scholes Partial Differential Equation and arbitrage arguments. They show that the value function follows a known differential equation which can be solved by a finite difference method. This approach is extended and generalized in Tanskanen and Lukkarinen (2004). They use a discretization method in order to solve the partial differential equation. Their model permits multiple distribution mechanisms, including those considered in Miltersen and Persson (2003) and Grosen and Jørgensen (2000).

However, these models can not be used to analyze some important features of contracts in insurance markets, where accounting rules allow for building and dissolving valuation reserves which can be used to stabilize the return on book values and, thus, the surplus distribution. In this case, insurers should consider the reserve quota when deciding how much surplus is distributed. Thus, the reserve situation is of great influence on the value of an insurance contract.

The present paper fills this gap: Surplus at time \( t \) can be determined and credited depending on the development of the assets (book or market value) and any management decision rule based on information available at time \( t \). Furthermore, minimum surplus distribution laws that exist in many countries may be considered. In particular, our model can represent all relevant features of the German market, including legal and supervisory issues as well as predominant management decision rules. On the other hand, our framework is general enough to include most of the above models and, therefore, products of other insurance markets as special cases.

We use a distribution mechanism that is typical for the German market which has been introduced in Kling et al. (2004). As opposed to their work, where the authors investigate, how the different parameters, such as the initial reserve quota, legal requirements, etc., affect the shortfall probability of a contract and how these factors interact, we are interested in the risk-neutral value of the corresponding contracts.

The rest of this paper is organized as follows. In Section 2 we introduce our model and the distribution mechanisms, i.e. the rules according to which earnings are distributed among policy holders, shareholders, and the insurance company. The goal of this paper is to find a fair price for an insurance policy using methods from financial mathematics. However, certain conditions must be fulfilled in order to obtain a “meaningful” price. In Section 3, we will discuss under which circumstances a risk-neutral valuation is appropriate. Since the considered insurance
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