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## A law of large numbers approach to valuation in life insurance

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#### Abstract

The classical Principle of Equivalence ensures that a life insurance company can accomplish that the mean balance per policy converges to zero almost surely for an increasing number of independent policyholders. By certain assumptions, this idea is adapted to the general case with stochastic financial markets. The implied minimum fair price of general life insurance policies is then uniquely determined by the product of the assumed unique equivalent martingale measure of the financial market with the physical measure for the biometric risks. The approach is compared with existing related results. Numeric examples are given. © 2006 Elsevier B.V. All rights reserved.

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#### 1. Introduction

Roughly speaking, the Principle of Equivalence of traditional life insurance mathematics states that premiums should be calculated such that incomes and losses are "balanced in the mean". Under the assumption that financial markets are deterministic, this idea leads to a valuation method usually called "Expectation Principle". The use of the two principles ensures that a life office can accomplish that (i.e. can buy hedges such that) the mean balance per policy converges to zero almost surely for an increasing number of policyholders. This is often referred to as the ability to "diversify" mortality (or biometric) risks. The main mathematical ingredients for this diversification are the stochastic independence of individual lives and the Strong Law of Large Numbers (SLLN). To obtain the mentioned convergence, it is neither necessary to have identical policies, nor to have i.i.d. lifetimes.

In modern life insurance mathematics, where financial markets are assumed to be stochastic and where more general products (e.g. unit-linked ones) are taken into consideration, the widely accepted valuation principle is an expectation principle, too. However, the respective probability measure is different since the minimum fair price

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or market value of an insurance claim is determined by the no-arbitrage pricing method known from financial mathematics. The respective equivalent martingale measure (EMM) is the product of the given EMM of the financial market with the physical measure for the biometric risks. Throughout the paper, we will call this kind of valuation the *product measure principle*. Although the result is not as straightforward as in the traditional case, a convergence property similar to the one mentioned above can be shown. So, diversification of biometric risks is still possible in the presence of stochastic financial markets, where payments related to e.g. unit-linked life policies of different policyholders may not be independent.

The aim of this paper is the derivation of an equivalent martingale measure for the pricing of life insurance policies starting from the assumption that, under the induced valuation principle, diversification of biometric risks should be possible by means of a convergence property as above, i.e. a life insurance company should be able to accomplish that the mean balance per contract converges to zero almost surely for an increasing number of independent policyholders. We will see that, under certain assumptions, the EMM then is uniquely determined and given by the product measure mentioned earlier, i.e. by the product of the given EMM of the financial market with the physical measure for the biometric risks. In different versions, diversification approaches have appeared in the literature on valuation. Considered as somehow straightforward, they are usually stated without proofs and for identical policies and i.i.d. lives, only. However, the derivation of a unique equivalent martingale measure and respective convergence properties for varying types of policies and lives at the same time, as carried out in this paper, needs a formally different setup and different mathematical tools than the derivation of a unique pricing rule for infinitely many *identical* policies for i.i.d. lives, as done in some papers. In this sense, the present paper has a technical focus. Particular emphasis is put on mathematically rigorous and explicit model assumptions necessary for the derivation of the mentioned results. For instance, we state integrability conditions for cash flows of not necessarily identical policies that are sufficient for the application of the SLLN even if independence gets lost by common financial risks.

Research on the valuation of unit-linked life insurance products already started in the late 1960s. One of the first results that was in its core identical to the product measure principle was Brennan and Schwartz (1976). In this paper, the authors "eliminate mortality risk" by assuming an "average purchaser of a policy", which clearly is a diversification argument. More recent papers mainly dedicated to valuation following this approach are Aase and Persson (1994) for the Black–Scholes model and Persson (1998) for a stochastic interest rate model. Aase and Persson (1994), but also other authors, a priori suppose independence of financial and biometric events. In their paper, an arbitrage-free and complete financial market ensures the uniqueness of the financial EMM. The product measure principle is here motivated by a diversification argument, but also by "risk neutrality" of the insurer with respect to biometric risks (cf. Aase and Persson (1994) and Persson (1998)). A more detailed history of valuation in (life) insurance can be found in Møller (2002), see also the references therein.

There exist other derivations of the product measure principle which do not rely on diversification arguments. In Møller (2001), for example, the product measure coincides with the so-called minimal martingale measure (cf. Schweizer (1995)). The works Møller (2002, 2003a,b) also consider valuation, but focus on hedging (mainly quadratic criteria), respectively advanced premium principles. Becherer (2003) uses exponential utility functions to derive prices of contracts. In an example for a certain type of contract for i.i.d. lives, he shows that the product measure principle evolves in the limit for infinitely many policyholders. In general, no-arbitrage pricing of insurance cash flows using martingales and equivalent martingale measures was introduced by Delbaen and Haezendonck (1989) and Sondermann (1991). Later, Steffensen (2000) described possible sets of price operators for life insurance contracts by respective sets of equivalent martingale measures. A more detailed discussion of some valuation approaches, among them Steffensen (2000) and Becherer (2003), will take place in Section 8.

The present paper works with a discrete finite time framework. Like other papers in this field, it is general in the sense that it does not propose particular models for the dynamics of financial securities or biometric events. The concept of a life insurance policy is introduced in a very general way and the presented methods are not restricted to particular types of contracts. The diversification approach is carried out by assuming certain properties (most of them also assumed in the articles cited above) of the underlying stochastic model, like e.g. independence of individuals, independence of biometric and financial events, no-arbitrage pricing, etc. To be able to model a wide variety of possible types of policies and lives, we assume an infinite product space for the biometric risks that also provides for each possible life (of which we may have infinitely many) infinitely many i.i.d. ones (= large cohorts of similar lives). In fact, the setting is that we consider biometric probability spaces (= lives) and random variables on their

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