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## A least-squares approach to fuzzy linear regression analysis

Pierpaolo D'Urso, Tommaso Gastaldi\*

Dipartimento di Statistica, Probabilità e Statistiche Applicate, Università degli Studi di Roma "La Sapienza", P.le Aldo Moro, 5, 00185 Roma, Italy

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## Abstract

This paper deals with a new approach to fuzzy linear regression analysis. A *doubly linear adaptive fuzzy regression model* is proposed, based on two linear models: a *core regression model* and a *spread regression model*. The first one "explains" the centers of the fuzzy observations, while the second one is for their spreads. As dependence between centers and spreads is often encountered in real world applications, our model is defined in such a way as to take into account a possible linear relationship among centers and spreads. Illustrative examples are also discussed, and a computer program which implements our procedure is enclosed. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Fuzzy data; Doubly linear adaptive fuzzy regression model; Least-squares estimators

## 1. Introduction

Vague or fuzzy data find application in several fields, such as psychometry, reliability, marketing, quality control, ballistics, ergonomy, image recognition, artificial intelligence, etc. A typical problem where vague data arise is that of assigning numbers to subjective perceptions or to linguistic variables (such as "enough", "good", "sufficiently", etc.). In fact, there are many cases where observations cannot be known or quantified exactly, and, thus, we can only provide an approximate description of them, or intervals to enclose them. For instance, "in measuring the influence of character size on the reading ability from a video display terminal [...] the reading ability of the subject, which is essentially the experimental output, depends on

\* Corresponding author.

E-mail addresses: durso@pow2.sta.uniromal.it (P. D'Urso), gastaldi@pow2.sta.uniromal.it (T. Gastaldi).



Fig. 1. Triangular membership function.

his/her eyesight, age, the environment, individual responses, and even how tired is the individual. Some of these factors cannot be described accurately and [...] the best description of these kinds of output is that they are fuzzy outputs" (Chang et al., 1996).

In the analysis of fuzzy data, the study of the relationship between a set of (crisp or fuzzy) independent variables and one or more dependent fuzzy variables is an important issue. Such a relationship can be studied, for instance, through fuzzy linear regression techniques. In particular, fuzzy linear regression was first introduced by Tanaka et al. (1980,1982). Several developments followed these pioneer papers, such as those studied by Chang and Lee (1994a–c, 1996), Diamond (1990), Diamond and Kloeden (1994), Kacprzyk and Fedrizzi (1992), Kim et al. (1996), Moskowitz and Kim (1993), Peters (1994), Redden and Woodall (1994, 1996), Savic and Pedrycz (1991), Tanaka (1987), Tanaka and Watada (1988), Tanaka and Ishibuchi (1991), Tanaka et al. (1995), Xizhao and Minghu (1992). Also several applications have been proposed: for instance, see Chang et al. (1996), Lee (1996), McCauley and Wang (1997).

Following Dubois and Prade (1980) and Zimmermann (1985), a fuzzy number may be defined as  $F = (\alpha, \beta, \gamma)_{LR}$ , where  $\alpha$  denotes the *center* (or *mode*),  $\beta$  and  $\gamma$  are the left *spread* (or *width*) and right spread, respectively, L and R denote the left and right *shape functions*. When  $\beta = \gamma$  and the shape functions are specular (w.r.t. the center), we have a *symmetrical* fuzzy number, denoted by  $F^* = (\alpha, \beta)$ . It is common to define a symmetrical fuzzy number by using a *triangular membership function* (also shown in Fig. 1):

$$\mu_{F^*}(u) = \begin{cases} 1 - \frac{|\alpha - u|}{\beta} & \text{if } \alpha - \beta \le u \le \alpha + \beta, \\ 0 & \text{otherwise.} \end{cases}$$

In the following section, we propose a new approach to fuzzy linear regression analysis. Our treatment is not based on the usual linear programming techniques, but upon the idea of fitting a regression model through minimization of a *distance function* between the empirical fuzzy data and the model.

For the sake of simplicity, we will consider the regression problem from a descriptive point of view, thus avoiding inferential issues.

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