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A sensitivity analysis of typical life insurance contracts with respect to the technical basis

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Abstract

In [Christiansen, M.C., 2007. A sensitivity analysis concept for life insurance with respect to a valuation basis of infinite dimension. Insurance: Math. Econom. doi:10.1016/j.insmatheco.2007.07.005] a sensitivity analysis concept was introduced for the prospective reserve of individual life insurance contracts as functional of the technical basis parameters such as interest rate, mortality probability, disability probability, et cetera. On the basis of that concept, the present paper gives in addition the sensitivities of the premium level.

Applying these approaches, an extensive sensitivity analysis is carried out: A study of the basic life insurance contract types ‘pure endowment insurance’, ‘temporary life insurance’, ‘annuity insurance’ and ‘disability insurance’ identifies their diverse characteristics, in particular their weakest points concerning fluctuations of the technical basis. An investigation of combinations of these insurance contract types shows what synergy effects can be expected by creating insurance packages.

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1. Introduction

The past has shown that actuarial assumptions such as interest rate or mortality can vary significantly within a contract period. Especially in recent years financial markets have shown a high volatility, and life expectancies in many developed countries increased with an unforeseen rate. Hence, an actuary is well advised to pay attention to the influence such changes can have on premiums or reserves.

In Christiansen (2007) a detailed overview is given of the literature addressing the dependency of prospective reserve or premium level on actuarial assumptions:

First, there are Lidstone (1905) and his successors Norberg (1985), Hoem (1988), Ramlau-Hansen (1988), and Linnemann (1993), who mainly presented qualitative statements. Second, there are scenario-based approaches, as for example Olivieri (2001) or Khalaf-Allah et al. (2006), but that idea works only for a small number of parameters.

A third way is to study sensitivities by means of derivatives, which turned out to be a very efficient concept. References using such an approach are Dienst (1995), Bowers et al. (1997), Kalashnikov and Norberg (2003), and Helwich (2003). All of those studies have in common that they only allowed for a finite number of parameters. In Christiansen (2007) a generalized gradient vector concept was introduced that enables to study sensitivities with respect to an infinite number of parameters. This meets, for example, the more realistic idea of actuarial assumptions (e.g. the mortality) being functions on a real line rather than on a discrete time grid.

Christiansen (2007) gave a general formula for the sensitivity of the prospective reserve of an individual insurance contract with respect to changes of the technical basis. The present paper gives in Section 2.2 an analogous formula for the sensitivity of the premium level of a contract. Applying them, Section 3 carries out a sensitivity analysis for various examples

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of typical life insurance contracts. It turns out that the different contract types have very diverse characteristics. With the calculated sensitivities being linear in the payment streams, the sensitivities of combinations of different insurance contracts are obtained by just adding up their sensitivities, which enables to easily create cancellation effects in order to reduce sensitivities. This simplicity makes the approach presented herein a valuable tool for the designing of insurance packages.

2. Life insurance model

The modelling framework used here follows the outline of Christiansen (2007). It is based on the continuous time Markov model launched by Hoem (1969) but extended to the approach based on cumulative intensities along the lines of Milbrodt and Stracke (1997).

Consider an insurance policy that is issued at time 0, terminates at a fixed finite time T, and is driven by a Markovian jump process $(X_t)_{t\in[0,T]}$ with finite state space $\mathcal{S}$. Let $J := \{(y,z) \in \mathcal{S}^2 | y \neq z\}$ denote the transition space. Since the state space $\mathcal{S}$ is finite, there exists a finite-dimensional *transition probability matrix* p,

$$p_{yz}(s,t) = P(X_t = z | X_s = y),$$
$$(y,z) \in \mathcal{S}^2, s \le t, P(X_s = y) > 0,$$

for which the Chapman–Kolmogorov equations hold. It corresponds to a so-called *cumulative transition intensity matrix* q. Assume that its nondiagonal entries q_{yz} are nondecreasing cadlag (right-continuous with left-hand limits) functions, zero at time zero, and the diagonal entries satisfy $q_{yy} = -\sum_{z:z\neq y} q_{yz}$, and $\Delta q_{yy}(t) \ge -1$ for all t. Then p can be calculated out of q via product integration (cf. Andersen et al. (1991, Section II.6.))

$$p(s,t) = \prod_{(s,t]} (\mathbb{I} + \mathrm{d}q) \tag{2.1}$$

$$:= \lim_{\max|t_i - t_{i-1}| \to 0} \prod (\mathbb{I} + q(t_i) - q(t_{i-1})) \tag{2.1}$$

for partitions $s < t_0 < t_1 < \cdots < t_n = t$.

Remark 2.1 (*Notation*). In the following denote by BV the linear space of functions on $\mathbb{R}$ with finite total variation and support in $[0,\infty)$. It gets a normed space by choosing the total variation as its norm. An additional subscript of '$\leftarrow$' means 'right-continuous'.

Payments between insurer and policyholder are of two types:

(a) Annuity payments fall due during sojourns in a state, modeled in a cumulative manner via the functions $B_z \in BV_{\leftarrow}$, $z \in \mathcal{S}$. (If the policy stays during the time interval $(s,t]$ in state z, $B_z(t) - B_z(s)$ is the accumulated amount falling due.) Benefits paid to the insured get a positive sign, premiums paid by the insured get a negative sign.
(b) Lump sums are payable upon a transition $(y,z) \in J$ between two states and are specified by the functions $b_{yz} \in BV$. To distinguish between the time of transition and the actual time of payment, define the function $DT : (0,\infty) \to (0,\infty)$, $DT(t) \ge t$, such that upon transition from y to z at time t the amount $b_{yz}(t)$ is payable at time $DT(t)$.

All payments are movements on an account that bears interest, which is here modelled by the *accumulation factor* $K : [0,\infty) \to (0,\infty)$. (A payment of one at time zero has at time t the value $K(t)$.) It shall be representable by

$$K(t) = \prod_{(0,t]} (1 + \mathrm{d}\Phi) \tag{2.2}$$

$$= \mathrm{e}^{\Phi(t) - \sum_{\tau \le t} \Delta\Phi(\tau)} \prod_{\tau\in(0,t]} (1 + \Delta\Phi(\tau)) \tag{2.2}$$

(cf. (2.6.2) in Andersen et al. (1991)), where Φ is the so-called *cumulative interest intensity* satisfying

$$\Phi \in BV_{\leftarrow}, \quad \Delta\Phi(t) := \Phi(t) - \Phi(t-) \ge C_\Phi > -1$$
$$\text{for all } t \ge 0. \tag{2.3}$$

2.1. Prospective reserve

Let DB_s be the present value at time s of all transition benefits triggered strictly after s, and let SB_s be the present value at time s of all payments falling due during sojourns in a state at and after time s. The (total) present value at time s is $SB_s + DB_s$.

Because of the finite time horizon $T < \infty$, the following integrability conditions hold always true,

$$\sum_{z\in\mathcal{S}} \int_{[0,T]} \frac{1}{K(t)} |B_z|(\mathrm{d}t) < \infty, \tag{2.4}$$

$$\sum_{(z,\zeta)\in J} \int_{(0,T]} \frac{1}{K(DT(t))} b_{z\zeta}(t) q_{z\zeta}(\mathrm{d}t) < \infty. \tag{2.5}$$

Denote

$$V_{y,s} := \mathbb{E}(SB_s + DB_s | X_s = y) \tag{2.6}$$

as the *prospective reserve at time* $s \ge 0$ *in state* $y \in \mathcal{S}$. One has

$$V_{y,s} = \sum_{z\in\mathcal{S}} \int_{[s,T]} \frac{K(s)}{K(t)} p_{yz}(s,t) B_z(\mathrm{d}t) + \sum_{(z,\zeta)\in J} \int_{(s,T]} \frac{K(s)}{K(DT(t))} b_{z\zeta}(t) p_{yz}(s,t-) q_{z\zeta}(\mathrm{d}t), \tag{2.7}$$

for all $s \in [0,T]$ and $y \in \mathcal{S}$ with $P(X_s = y) > 0$ (see Milbrodt and Stracke (1997), Lemma 4.4.). Following Christiansen (2007), let the cumulative interest intensity and the cumulative transition intensities decompose to

$$\Phi + H_\Phi \in BV_{\leftarrow}, \qquad q_{yz} + H_{yz} \in BV_{\leftarrow}, \quad (y,z) \in J, \tag{2.8}$$

where Φ and $q_J := (q_{yz})_{(y,z)\in J}$ build an arbitrary but fixed 'initial point', which varies with 'deviation' $(H_\Phi, H_J) := (H_\Phi, (H_{yz})_{(y,z)\in J})$. Define

$$BV_{\leftarrow,-1} := \{F \in BV_{\leftarrow} | \Delta F(t) \ge C_F > -1, \quad \text{for all } t \in \mathbb{R}\} \tag{2.9}$$