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Fuzzy least-squares linear regression analysis using shape preserving operations

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Abstract

In this paper, we introduce a new class of fuzzy linear regression model with fuzzy input – fuzzy output data using shape preserving arithmetic operations on *L–R* fuzzy numbers for least-squares fitting. A mixed nonlinear programming approach is used to derive the satisfying solution. This answers one of Diamond's questions [Inform. Sci. 46 (1988) 141]. © 2001 Elsevier Science Inc. All rights reserved.

1. Introduction

Since Tanaka et al. in 1982 [11] proposed a study in linear regression analysis with fuzzy model, the fuzzy regression analysis has been widely studied and applied in a variety of substantive areas. A collection of recent papers dealing with several approaches to fuzzy regression analysis can be found in [7]. In this paper, we concentrate on the model of Diamond [2]. Diamond [2] proposed the so-called fuzzy least-squares. Recently, Diamond's method has been revisited in [3].

Let $X = (m, \alpha, \beta)$ be a triangular fuzzy number where m is the modal value of X and α and β are the left and right spreads, respectively. Diamond [2] gave a metric d on the space $\mathscr{T}(R)$ of all triangular fuzzy numbers by

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$$d(X,Y)^2 = (m_X - m_Y)^2 + ((m_X - \alpha_X) - (m_Y - \alpha_Y))^2 + ((m_X + \beta_X) - (m_Y + \beta_Y))^2,$$

where $X = (m_X, \alpha_X, \beta_X)$ and $Y = (m_Y, \alpha_Y, \beta_Y)$ are any two triangular fuzzy numbers in $\mathscr{T}(R)$. A linear structure is defined on $\mathscr{T}(R)$ by $(m_X, \alpha_X, \beta_X) + (m_Y, \alpha_Y, \beta_Y) = (m_X + m_Y, \alpha_X + \alpha_Y, \beta_X + \beta_Y)$, $t(m, \alpha, \beta) = (tm, t\alpha, t\beta)$ if $t \geqslant 0$, $t(m, \alpha, \beta) = (tm, |t|\beta, |t|\alpha)$ if $t < 0$.

There are three simple fuzzy regression models considered in [2]:

$$\text{(F1):} \quad Y = a + bX, \quad a, b \in \mathscr{R}, \ X \in \mathscr{T}(R),$$
$$\text{(F2):} \quad Y = E + bX, \quad b \in \mathscr{R}, \ E, X \in \mathscr{T}(R),$$
$$\text{(F3):} \quad Y = A + Bx, \quad x \in \mathscr{R}, \ A, B \in \mathscr{T}(R).$$

The corresponding least-squares optimization problems are:

$$\text{(M1):} \quad \text{minimize } r(a,b) = \sum_{i=1}^{n} d(a + bX_i, Y_i)^2,$$
$$\text{(M2):} \quad \text{minimize } r(E,b) = \sum_{i=1}^{n} d(E + bX_i, Y_i)^2,$$
$$\text{(M3):} \quad \text{minimize } r(A,B) = \sum_{i=1}^{n} d(A + Bx_i, Y_i)^2.$$

The models are rigorously justified by a projection-type theorem for cones on a Banach space containing the cone of triangular fuzzy numbers. In conclusion of [2], Diamond mentioned about the following model:

$$\text{(F4):} \quad Y = A + BX,$$

where A, B, and X are fuzzy numbers and BX is fuzzy multiplication. If (X_i, Y_i) are received in the form of triangular fuzzy numbers, such a fit is neither possible, nor would it make good sense. This is because multiplication BX results in membership functions with drumlike sides.

In 1992, Bardossy et al. [1] defined a new class of distance on *L–R* fuzzy numbers and considered the model (F4). Recently, Hong and Do [5] introduced a shape preserving operations based on sup-t-norm convolutions in both addition and multiplication and Hong et al. [6] presented a new method to evaluate fuzzy linear regression models based on Tanaka's approach, where both input data and output data are fuzzy numbers, using T_W-based fuzzy arithmetic operations. Actually, T_W (the weakest t-norm) based fuzzy arithmetic operations is the only shape preserving operations in both addition and multiplication [4].

In this paper, we apply shape preserving arithmetic operations based on the T_W (the weakest t-norm) convolution in both addition and multiplication and