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Optimal weighted average power similar tests for the covariance structure in the linear regression model

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Abstract

This paper suggests a procedure for the construction of optimal weighted average power similar tests for the error covariance matrix of a Gaussian linear regression model when the alternative model belongs to the exponential family. The paper uses a saddlepoint approximation to construct simple test statistics for a large class of problems and overcomes the computational burden of evaluating the complicated integrals arising in the derivation of optimal weighted average power tests. Extensions to panel data models are considered. Applications are given to tests for error autocorrelation in the linear regression model and in a panel data framework.

JEL classification: C12; C21; C22; C23

Keywords: Linear model; Similar tests; Covariance matrix; Optimal weighted average tests

1. Introduction

Many hypothesis testing problems in the Gaussian linear regression model take the form of tests on the covariance matrix of the errors. Among them there are tests for the presence of serial correlation (Anderson, 1948; Durbin and Watson, 1950; Godfrey, 1978; King and Hillier, 1985; King, 1988; Dufour and King, 1991), tests for a unit root (Sargan and Bhargava, 1983; Elliott et al., 1996), tests for heteroskedasticity of various forms (Goldfeld and Quandt, 1965; Glejser, 1969; Breusch and Pagan, 1979; King and Hillier, 1985; Pagan and Pak, 1992), and tests for outliers (Joshi and Lalitha, 1986). The literature for each of these testing problems is extensive.

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It is well known that in general there are no uniformly most powerful invariant/similar tests in these situations, and alternative optimality criteria must be developed. For example, when testing for the presence of autocorrelation, Anderson (1948) has argued that a good test can be constructed by approximating the testing set-up with another one admitting a uniformly most powerful test (this procedure has been later used by Durbin and Watson, 1950; Sargan and Bhargava, 1983). King and Hillier (1985) have obtained locally best invariant tests under a wide class of alternatives. Their work has been extended by Dufour and King (1991) for the case of autocorrelated errors. King (1988) has given an excellent review of the use of point optimal tests, i.e. tests which are optimal against a specific alternative, and has argued that a careful selection of this specific alternative can lead to good tests. Elliott et al. (1996) have also recommended the construction of point optimal tests when testing for a unit root in the errors.

We follow the suggestion of Wald (1943), Hillier (1987) and Andrews and Ploberger (1994) and construct tests for the covariance matrix of a normal linear regression model, which are optimal in terms of a weighted average power (WAP). The main problem with the construction of these tests is that complicated (possibly multivariate) integrals need to be evaluated. However, given some assumptions, these integrals can be approximated with the saddlepoint method, and the resulting statistics have simple functional forms.

This approach has a few advantages over existing methodologies. Firstly, it is based on classical statistical principles, and delivers tests which are optimal (given the assumptions made) according to well known optimality criteria. Secondly, it yields as limiting cases most of the existing tests (point optimal tests, locally best invariant and locally best unbiased invariant tests). Thirdly, it can easily deal with the case in which the alternative is described by more than one parameter, and with "two-sided" alternatives. Fourthly, it delivers tests as powerful as tests satisfying other optimality criteria when these are available. This suggests that our approach will lead to good tests in other situations where it can be applied.

The testing set-up and the first main result (Theorem 2) are discussed in Section 2. Here, the assumption of normality allows us to simplify the analysis, even though the results obtained can be indicative for more general models. For the sake of simplicity, we also assume that the model belongs to the exponential family under the alternative hypothesis, although this assumption can be relaxed. Section 2.1 compares the optimal WAP test with existing tests for autocorrelation of the error terms, and shows that the optimal WAP tests have powers close to the Durbin and Watson (1950) test. They can be very different, however, from the power of the Lagrange Multiplier test of Godfrey (1978) and Breusch and Pagan (1980) in small samples. In Section 2.2 we also reformulate the problem to test for a unit root in the errors and show that the WAP tests significantly improve on the test suggested by Sargan and Bhargava (1983). Section 3 extends the approximation for the optimal WAP tests to panel data models and the second main result (Theorem 3) is given. We show that the optimal weighed average invariant tests are the same for the fixed effects and the random effects model, provided that we impose invariance to some groups of transformations. Section 3.1 gives a comparison with the